Parallel Execution of Logic Programs
A Tutorial
(Or: Multicores are here! Now, what do we do with them?)

Manuel Hermenegildo

IMDEA Software
Tech. University of Madrid
U. of New Mexico

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Introduction / Motivation

- **Multicore chips** have moved parallelism from niche (HPC) to mainstream – even on laptops!

- According to vendors (and Intel in particular [e.g., DAMP workshops]):
  - Feature size reductions will continue for foreseeable future (12 generations!).
  - But power consumption does not allow increasing clock speeds much.
  - Multicore is the way to use this space without raising power consumption.
  - Number of cores expected to *double* with each generation!

- But writing parallel programs hard/error-prone – how to exploit all those cores?
  - Ideal situation: *Conventional Program + Multiprocessor = Higher Perf.*
    → automatic parallelization.
  - More realistically: compiler-*aided* parallelization.
  - Languages (dialects, constructs) for parallelization+parallel programming.
  - Scheduling techniques, memory management, abstract machines, etc.
LP and CLP – quite interesting from the parallelism point of view

- Many parallelism-friendly aspects:
  - program close to problem description $\rightarrow$ less hiding of intrinsic parallelism
  - well understood mathematical foundation $\rightarrow$ simplifies formal treatment
  - relative purity (well behaved variable scoping, fewer side-effects, generally single assignment) $\rightarrow$ more amenable to automatic parallelization.

- At the same time, requires dealing with the most complex problems:
  - irregular computations; complex data structures; (well behaved) pointers;
    dynamic memory management; recursion; ...

patterns, but in a much more elegant context;
and brings up some upcoming issues (e.g., speculation, search, constraints).

$\rightarrow$ Very good platform for developing universally useful techniques:
Examples to date: conditional dep. graphs, abstract interpretation w/interesting domains, cost analysis / gran. control, dynamic sched. and load balancing, ...
Complex Data Structures / Pointers

- Example:

  \[
  \text{main :- } X = f(Y,Z),
  \]

  \[
  Y = a,
  \]

  \[
  W = Z,
  \]

  \[
  W = g(K),
  \]

  \[
  X = f(a,g(b)).
  \]
Parallelism in Logic Programs and CLP

- **Or-parallelism**: execute simultaneously different search space branches.
  - Present in general search problems, enumeration part of constr. problems, etc.
    
    ```prolog
    money(S,E,N,D,M,O,R,Y) :-
        digit(0),
        digit(S),
        digit(E),
        ..., carry(I),
        ..., N is E+O-10*I,
        carry(0),
        carry(1).
    ```

- **And-parallelism**: execute simultaneously different clause body goals.
  - Comprises traditional parallelism (parallel loops, divide and conquer, etc.).
  - *Concurrent languages* also generally based on and-parallelism.
    
    ```prolog
    qsort([X|L],R) :-
        partition(L,X,L1,L2),
        qsort(L2,R2),
        qsort(L1,R1),
        append(R1,[X|R2],R).
    ```
Objective and Issues

- **Temptation:** make use of all this potential.

- **Problem:** this can yield a slowdown or even erroneous results.

- **Objective**: and/or-parallel execution of (some of the goals in) logic programs (and full Prolog, CLP, CC, ...), while:
  - obtaining the same solutions as the sequential execution (i.e., correctness)
  - taking a shorter or equal execution time (speedup or, at least, no-slowdown over state-of-the-art sequential systems) (i.e., efficiency).

- Above conditions may not always be met:
  - Independence: conditions that the run-time behavior of the goals must satisfy to guarantee correctness and efficiency (under ideal conditions – no overhead).

- The presence of overheads complicates things further:
  - Granularity Control: techniques for ensuring efficiency in the presence of overheads.
Sequential and Parallel Execution Framework: OR

- Model: consider a state $G = \langle g_1 : g_2 : \ldots : g_n, \theta \rangle$ where we select $g_1$.

- If there are two clauses:
  
  \[
  g_1' \leftarrow g_{11}', \ldots, g_{1m}'. \\
  g_1 \leftarrow g_{11}, \ldots, g_{1k}.
  \]

- We construct two states:
  
  \[
  G' = \langle g_{11}' : \ldots : g_{1m}' : g_2 : \ldots : g_n, \theta' \theta' \rangle \\
  G'' = \langle g_{11}'' : \ldots : g_{1k}'' : g_2 : \ldots : g_n, \theta'\theta'' \rangle
  \]

- Sequential execution: execute $G'$ first and then $G''$.

- Parallel execution: execute $G'$ and $G''$ in parallel.

- Since $G'$ and $G''$ are completely independent:
  
  ◦ Same results are obtained in parallel or sequentially.
  ◦ All branches can be explored in parallel.
  ◦ Same number of branches explored (only if “all sols”!).

- Thus, or-parallelism: mostly implementation issues.
  
  (but side-effects, cuts, and aggregation predicates complicate things)
Issues in OR Parallelism

- System organization:
  - System comprises a collection of **workers** (processes/processors).
  - Each worker is an LP/CLP engine with a full set of stacks.
  - A **scheduler** assigns unexplored branches to idle workers.

- Main implementation problem: alternative bindings – efficiently maintaining different environments per branch (e.g., $p_1$ and $p_2$ in example):
  - Sharing (e.g. **Aurora**, PEPSys/ECLIPSE, etc.)
  - Recomputation (e.g. **Delphi** model)
  - Copying (e.g. **Muse system**) ECLIPSE, SICStus, OZ).
  - Theoretical limitations. Desirable:
    * Constant–time access to variables
    * Constant–time task creation
    * Constant–time task switching

Impossible to meet all three with a finite number of processors.
(Hence, they are not met in sequential execution!)
Issues in Or-parallelism: Illustration

..., p(X), ...

\[ p_1(X) \leftarrow ... , X=a , ... , ! , ... \]
\[ p_2(X) \leftarrow ... , X=b , ... \]

\[ \text{main} \leftarrow l , s \]
\[ \quad : \parallel \ l/0. \]
\[ \quad l \leftarrow \text{large}_\text{work}_a . \]
\[ \quad l \leftarrow \text{large}_\text{work}_b . \]
\[ \quad : \parallel \ s/0. \]
\[ \quad s \leftarrow \text{small}_\text{work}_a . \]
\[ \quad s \leftarrow \text{small}_\text{work}_b . \]
Issues in OR Parallelism

- Speculation (e.g., \( p_2 \) in example).
  - To guarantee speedup: avoid speculative work – too strong/difficult?
  - To guarantee no-slowdown:
    * Left-biased scheduling.
    * Instantaneous killing on cut.

- Granularity: avoid parallelizing work that is too small.

- Parallelization can be done:
  - Adding `parallel/1` annotations to selected predicates (ANL,ECLIPSE)
  - Others (Aurora, MUSE) automatically via the scheduler.

- Useful supporting techniques identified:
  - Visualization/trace analysis: ANL, VisAndOr/IDRA, ViMust, Parsee, VisAll, ...
  - Program transformation to increase granularity
  - Compile-time/run-time granularity control; automatically introduce `parallel` annotations
Some Results in OR Parallelism

- Quite successful systems built (ECLIPSE, SICSTUS/MUSE, Aurora, OrpYap, etc.)
- MUSE is quite easy to add to an existing Prolog system (done with Prolog by BIM, also added to SICStus Prolog V3.0)
- Significant speedups w.r.t. state-of-the-art Prolog systems can be obtained with Aurora and Muse for search-based applications.

<table>
<thead>
<tr>
<th>Program</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>10</th>
<th>Sicstus 0.6</th>
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<tbody>
<tr>
<td>parse1</td>
<td>1.8</td>
<td>2.8</td>
<td>2.93</td>
<td>2.76</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
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<td>3.74</td>
<td>6.92</td>
<td>7.72</td>
<td>1.27</td>
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<tr>
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<td>3.74</td>
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<td>7.34</td>
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<td>tina</td>
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<td>4.06</td>
<td>7.81</td>
<td>9.59</td>
<td>1.43</td>
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</tbody>
</table>

- Much work done on schedulers (left bias, cut, side effects, ....)
• Easy to extend to CLP (e.g., VanHentenryck, ECLIPSE system).
Simple Goal-level And-Parallel Exec. Framework

- Model:
  consider a state \( G = \langle g_1 : g_2 : \ldots : g_n, \theta \rangle \), to execute \( g_1 \) and \( g_2 \) in parallel:
  - execute \( \langle g_1, \theta \rangle \) and \( \langle g_2, \theta \rangle \) in parallel (fork) obtaining \( \theta_1 \) and \( \theta_2 \),
  - continue with \( \langle g_3 : \ldots : g_n, \theta_1 \theta_2 \rangle \) (join).

- Regarding multiple solutions – two possibilities:
  - Gather all solutions for both goals separately.
  - Perform “parallel backtracking”.

- Multiple problems, related to variable binding conflicts: during parallel execution of \( \langle g_1, \theta \rangle \) and \( \langle g_2, \theta \rangle \) the same variable may be bound to inconsistent values.

- Correctness problems (due to the definition of composition of substitutions – e.g. \( x/a \) composed with \( x/b \) succeeds!)
  Solutions (proved correct in case of “pure” goals):
  - Modify definition of composition: \( \theta \circ \eta(t) = \text{mgu}(E(\theta) \cup E(\eta))(t) \)
  - Change parallel model.
  - Not an issue in CLP: conjunction instead of composition.
Issues in And-Parallelism – Independence

- **Correctness:** “same” solutions as sequential execution.
- **Efficiency:** execution time < than seq. program (or, at least, *no-slowdown*: \( \leq \)).
  (We assume parallel execution has no overhead in this first stage.)

- Running at \( s_2 \) “seeing \( s_1 \)”: 

<table>
<thead>
<tr>
<th></th>
<th>Imperative</th>
<th>Functions</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_1 )</td>
<td>( Y := W+2; )</td>
<td>((+ W 2))</td>
<td>( Y = W+2,)</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>( X := Y+Z; )</td>
<td>((+ Z))</td>
<td>( X = Y+Z,)</td>
</tr>
<tr>
<td>read-write deps</td>
<td>strictness</td>
<td>cost!</td>
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<p>| |</p>
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<tbody>
<tr>
<td>( s_1 )</td>
</tr>
<tr>
<td>( s_2 )</td>
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<tr>
<td></td>
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</tbody>
</table>

For **Predicates** (multiple procedure definitions):

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<tbody>
<tr>
<td>( \text{main} :- ) ( p(X). )</td>
<td>( p(X) :- X=a. )</td>
</tr>
<tr>
<td>( s_1 )</td>
<td>( q(X). )</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>( \text{write}(X). )</td>
</tr>
</tbody>
</table>

Again, cost issue: *if* \( p \) affects \( q \) (prunes its choices) *then* \( q \) ahead of \( p \) is speculative.

- **Independence:** condition that guarantees correctness *and efficiency.*
Independence and its Detection

- Informal notion: a computation “does not affect” another (also referred to as “stability” in, e.g., EAM/AKL).

- Greatly clarified when put in terms of Search Space Preservation (SSP) – shown SSP sufficient and necessary condition for efficiency.

- Detection of independence:
  - Run-time (a-priori conditions)
  - Compile-time
  - Mixed: conditional execution graph expressions (1)
  - User control: explicit parallelism (concurrent languages). (2)

- (1)+(2) = &-Prolog: view parallelization as a source to source transformation of original program into a parallelized (“annotated”) one in a concurrent/parallel language. Allows:
  - Automatic parallelization — and understanding the result).
  - User parallelization — and the compiler checking it).
Concrete System Used in Examples: Ciao

- For concreteness, hereafter we use &-Prolog (now Ciao) as a target. The relevant minimal subset of &-Prolog/Ciao:
  - Prolog (with if-then-else, etc.).
  - Parallel conjunction “&/2” (with correct and complete forwards and backwards semantics).
  - A number of primitives for run-time testing of instantiation state.
- Ciao is one of the popular Prolog/CLP systems (supports ISO-Prolog fully).

Many other features: new-generation multi-paradigm language/prog.env. with:
  - Predicates, constraints, functions (including lazyness), higher-order, ...
  - Assertion language for expressing rich program properties (types, shapes, pointer aliasing, non-failure, determinacy, data sizes, cost, ...).
  - Static debugging, verification, program certification, PCC, ...
  - Parallel, concurrent, and distributed execution primitives.
    - Automatic parallelization.
    - Automatic granularity and resource control.
A Priori Independence: Strict Independence-I

- Approach (goal level). Consider parallelizing \( p(X,Y) \) and \( q(X,Z) \):

\[
\text{main :-}
\text{\quad t(X,Y,Z),}
\text{\quad s_1 p(X,Y),}
\text{\quad s_2 q(X,Z).}
\]

We compare the behaviour of \( s_2 q(X,Z) \) and \( s_1 q(X,Z) \).

- A-priori Independence: when reasoning only about \( s_1 \). Can be checked at run-time before execution of the goals.

- A priori independence in the Herbrand domain: Strict Independence [DeG84, HR89]: goals do not share variables at run-time.

- Example 1: Above, if \( t(X,Y,Z) :- X=a \).
A Priori Independence: Strict Independence-II

- The “pointers” view:

  correctness and efficiency (search space preservation) guaranteed for p & q if there are no “pointers” between p and q.

  main :- X=f(K,g(K)), Y=a, Z=g(L), W=h(b,L),

  --------------------->

  p(X,Y),
  q(Y,Z),
  r(W).

  p and q are strictly independent, but q and r are not.
A Priori Independence: Strict Independence-III

• **Example 2:**

\[
qs([X|L], R) :- \quad \text{part}(L, X, L1, L2), \\
\quad qs(L2, R2), \quad qs(L1, R1), \\
\quad \text{app}(R1, [X|R2], R).
\]

Might be annotated in &-Prolog (or Ciao) as:

\[
qs([X|L], R) :- \\
\quad \quad \text{part}(L, X, L1, L2), \\
\quad \quad (\text{indep}(L1, L2) \rightarrow qs(L2, R2) \& qs(L1, R1) \\
\quad \quad \quad ; \quad qs(L2, R2), \quad qs(L1, R1)), \\
\quad \quad \text{app}(R1, [X|R2], R).
\]

• Not always possible to determine locally/statically:

\[
\text{main} :- \quad \text{t}(X, Y), \quad \text{p}(X), \quad \text{q}(Y).
\]

\[
\text{main} :- \quad \text{read}([X, Y]), \quad \text{p}(X), \quad \text{q}(Y).
\]

• Alternatives: run-time independence tests, global analysis, ...
Fundamental issues:

- Can we build a system which obtains speedups w.r.t. a state of the art sequential LP system using such annotations?

- Can those annotations be generated automatically?
And-Parallelism Implementation

- By translation to or-parallelism:
  - Simplicity
  - Relatively high overhead $\rightarrow$ higher need for granularity control
  - Used, e.g., in ECLIPSE system.

- Direct implementation:
  - Abstract machine needs to be modified: e.g., PWAM / Marker model, EAM/AKL box machine.
    * System comprises a collection of agents (processes/processors).
    * Each agent is an LP/CLP engine with a full set of stacks.
    * Scheduling is normally done lazily through goal stacks.
  - Low overhead (but granularity control still useful)
  - Direct support for concurrent/parallel language
  - Used in &-Prolog/Ciao and most other systems: ACE, IDIOM, DDAS, ...

- Also, higher-level implementations (see later).
And-Parallelism Implementation

- Issues in direct implementation:
  - Scheduling / fast task startup.
  - Memory management.
  - Use of analysis information to improve indexing.
  - Local environment support.
  - Recomputation vs. copying.
  - Efficient implementation of parallel backtracking (and opportunities for intelligent backtracking).
  - Efficient implementation of “ask” (for communication among threads).
  - etc.
&-Prolog Run-time System: PWAM architecture

- Evolution of the RAP-WAM (the first Multisequential Model?) and Sicstus WAM.

- Defined as a storage model + an instruction set.
&-Prolog Run-time System: Agents and Stack Sets

- Agents separate from Stack Sets; Dynamic creation/deletion of S.Sets/Agents
- Lazy, on demand scheduling

Extensions / optimizations:
- DASWAM / DDAS System (dependent and-//)
- &ACE, ACE Systems (or-, and-, dep-//)
&-Prolog Run-time System: Performance

Sequent Symmetry, **hand parallelized** programs.
(Speedup over state of the art sequential systems.)
Visualization of And-parallelism – (small) qsort, 1 processor

(VisAndOr output.)
Visualization of And-parallelism – (small) qsort, 4 processors

(VisAndOr output.)
Independence – Strict Independence (Contd.)

- Not always possible to determine locally/statically:

  \[\text{main} :- \text{t}(X,Y), \text{p}(X), \text{q}(Y).\]

  \[\text{main} :- \text{read}([X,Y]), \text{p}(X), \text{q}(Y).\]

- Alternatives: run-time independence tests, global analysis, ...

  \[\text{main} :- \text{read}([X,Y]), ( \text{indep}(X,Y) \rightarrow \text{p}(X) \& \text{q}(Y) \]

  \[\quad \quad ; \quad \text{p}(X), \text{q}(Y) ).\]

  \[\text{main} :- \text{t}(X,Y), \text{p}(X) \& \text{q}(Y). \quad \%\% \ (\text{After analysis})\]
Parallelization Process: CDG-based Automatic Parallelization

- **Conditional Dependency Graph** (of some code segment):
  - Vertices: possible tasks (statements, calls, bindings, etc.).
  - Edges: possible dependencies (labels: conditions needed for independence).
- Local or global analysis used to reduce/remove checks in the edges.
- Annotation process converts graph back to parallel expressions in source.

```
foo(...) :-
g1(...),
g2(...),
g3(...).
```

**Figure:**
```
foo(...) :-
g1(...),
g2(...),
g3(...).
```

```
foo(...) :-
g1(...),
g2(...),
g3(...).
```

```
( test(1-3) -> ( g1, g2 ) & g3
; g1, ( g2 & g3 ) )
```

**Alternative:**
```
g1, ( g2 & g3 )
```
Simplifying Independence Conditions (Strict Ind.)

- Recall that $b_1$ and $b_2$ are strictly independent for $\theta$ iff

  $$\text{vars}(b_1\theta) \cap \text{vars}(b_2\theta) = \emptyset$$

- $\text{indep}(b_1, b_2)$ iff $b_1$ and $b_2$ do not share variables at run–time.

- $p(x, y)$ and $q(y, z)$ are strictly independent at run–time iff $\text{indep}(\{x, y\}, \{y, z\})$.

- Equivalent to $\{\text{indep}(x, y), \text{indep}(x, z), \text{indep}(y, y), \text{indep}(y, z)\}$.

- Domain of interpretation $DI$: subset of propositional logic.

- For clause $C$, it contains predicates of the form $\text{ground}(x)$ and $\text{indep}(y, z)$, $\{x, y, z\} \subseteq \text{vars}(C)$, with axioms:

  $\{\text{ground}(x) \rightarrow \text{indep}(x, y) | \{x, y\} \subseteq \text{vars}(C)\}$

  $\{\text{indep}(x, x) \rightarrow \text{ground}(x) | x \in \text{vars}(C)\}$

- The set $\{\text{indep}(x, y), \text{indep}(x, z), \text{indep}(y, y), \text{indep}(y, z)\}$ can be simplified to $\{\text{ground}(y), \text{indep}(x, z)\}$. 
Simplifying Independence Conditions (Strict Ind.)

Identify Dependencies

q(x,z) ->
\[ gnd(x) \]
\[ \text{ind}(y,z) \]
\[ \text{ind}(x,w) \]
p(x,y) ->
\[ \text{ind}(x,z), \text{ind}(x,w) \]
\[ \text{ind}(y,z), \text{ind}(y,w) \]
s(z,w)

Analysis Info

\[ \neg \text{gnd}(z) \]
\[ \text{ind}(y,z) \]
\[ \text{true} \]

Simplify Dependencies

p(x,y) ->
\[ \text{ind}(y,w) \]
s(z,w)

h(x,y,z):- \((p(x,y) \ & \ q(x,z)), s(z,w)\).

h(x,y,z):- \text{ind}(y,w) \rightarrow p(x,y) \ & \ (q(x,z), s(z,w)) \ ; p(x,y), q(x,z), s(z,w).
&-Prolog/Ciao Parallelizer Overview

USER

Prolog code

Annotators (local dependency analysis)
MEL/CDG/UDG/URLP/...

& – Prolog

&–Prolog system
(Parallel)

PARALLELIZING COMPILER

Abstract Interpretation
(Sharing, Sharing+Freeness, Aeqs, Def, Lsign, ...)

Dependency Info

side–effect analysis

granularity analysis
\textbf{&-Prolog/CIAO compiler overview (Contd.)}

Parallelizing compiler \cite{HW87} (now integrated in CiaoPP \cite{HBPLG99,HPBLG03}):

- Global Analysis: infers independence information.
- \textbf{Annotator(s): Prolog → &-Prolog parallelization} \cite{DeG87,MH90,BGH94a,CH94,PGPF97,MBdlBH99}.
  - MEL: Maximum Expression Length — simple heuristic.
  - CDG: Conditional Graph Expressions — graph partitioning of clauses.
  - UDG: Unconditional Graph Expressions.
  - Variants of CDG/UDG.
  - Enhanced to better use global analysis info and granularity information (still on–going).

- \textbf{Low-level PWAM compiler: extension of Sicstus V0.5}

- \textbf{Granularity Analysis: determines task size or size functions} \cite{DLH90,DL91,DL93,DLGHL94,DLGHL97,DLGH97,SCK98,MLGCH08}.

- \textbf{Granularity Control: restricts parallelism based on task sizes} \cite{DLH90,LGHD96,SCK98}.

- \textbf{Other modules: side effect analyzer} (sequencing of side-effects, coded in &-Prolog), \textit{multiple specializer / partial evaluator, invariant eliminator}, etc.
&-Prolog compilation: examples - I

multiply([],_,[]).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    vmul(V0,V1,Vr),
multiply(V0s,V1,Vrs).

vmul([],[],0).
vmul([H1|T1],[H2|T2],Vr) :-
    scalar_mult(H1,H2,H1xH2),
    vmul(T1,T2,T1xT2),
    Vr is H1xH2+T1xT2.

scalar_mult(H1,H2,H1xH2) :- H1xH2 is H1*H2.

Source (Prolog)
\begin{verbatim}
multiply([], _, []).
multiply([V0|V0s], V1, [Vr|Vrs]) :-
    ( ground([V1]), indep([[V0, V0s], [V0, Vrs], [V0s, Vr], [Vr, Vrs]])
    -> vmul(V0, V1, Vr) & multiply(V0s, V1, Vrs)
    ; vmul(V0, V1, Vr), multiply(V0s, V1, Vrs) ).

vmul([], [], 0).
vmul([H1|T1], [H2|T2], Vr) :-
    ( indep([[H1, T1], [H1, T2], [T1, H2], [H2, T2]])
    -> scalar_mult(H1, H2, H1\times H2) & vmul(T1, T2, T1\times T2)
    ; scalar_mult(H1, H2, H1\times H2), vmul(T1, T2, T1\times T2) ),
    Vr is H1\times H2 + T1\times T2.

scalar_mult(H1, H2, H1\times H2) :- H1\times H2 is H1\times H2.
\end{verbatim}

Parallelized program (&-Prolog/Ciao)—no global analysis
Dependency Analysis: Global Analysis Subsystem

- “PLAI” analyzer – top-down driven bottom up analysis (enhanced version of Bruynooghe’s scheme [Bru91]).

- Optimized fixpoint algorithm (keeps track of dependencies and approximation state of information, avoids recomputation) [MH89, HPMS00, PH96].

- Some useful abstract domains:
  ◦ Sharing Domain Abstraction (“S”) [MH89, LH92, MH92].
  ◦ Sharing+Freeness Domain Abstraction (“SF”) [MH91].
  ◦ Sondergaard’s $A_{Sub}$ (linearity) domain (“P”) [Søn86, MS93].
  ◦ Type domains, depth-K, etc.
  ◦ (Constraints:) Definiteness, Freeness, LSign domains.

- Domains combined using framework: e.g. $A_{Sub}+SH$, $A_{Sub}+ShF$

- Automatic elimination of repetitive checks [GH91, PH99].

- Current analyzer quite robust, with support for a relatively complete set of builtins.

- Support for full Prolog, CLP(R), etc.
“Sharing” Abstraction (Groundness + Set Sharing)

- **Definitions:**
  - $U_{var}$: universe of all variables,
  - $P_{var}$: set of program variables in a clause,
  - $Subst$: set of all possible mappings from variables in $P_{var}$ to terms.

- **Abstract Domain:** $D_\alpha = \wp(\wp(P_{var}))$

- **Abstraction of a substitution:**
  $$\alpha(A) : Subst \rightarrow D_\alpha$$
  $$\alpha(\theta) = \{Occ(\theta, U) | U \in U_{var}\} \text{ where } Occ(\theta, U) = \{X | X \in \text{dom}(\theta) \land U \in \text{var}(X\theta)\}.$$

- **Example:** Let $\theta = \{W = a, X = f(A_1, A_2), Y = g(A_2), Z = A_3\}$.
  $$\alpha(\theta) = \{\emptyset, \{X\}, \{X, Y\}, \{Z\}\}.$$

- **Note that**
  $$\diamond independent(x\theta, y\theta) \iff \nexists v \in U_{var}, x \in Occ(\theta, v) \land y \in Occ(\theta, v)$$

Other additional axioms are encoded in the sharing patterns.
:- entry multiply(g,g,f).

multiply([],_,[]).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    multiply(V0s,V1,Vrs),
    vmul(V0,V1,Vr).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    multiply(V0s,V1,Vrs),
    vmul(V0,V1,Vr).

vmul([],[],0).
vmul([H1|T1],[H2|T2],Vr) :-
    scalar_mult(H1,H2,H1xH2),
    vmul(T1,T2,T1xT2),
    Vr is H1xH2+T1xT2.

scalar_mult(H1,H2,H1xH2) :-
    H1xH2 is H1*H2.

Sharing information inferred by the analyzer
multiply([],_,[]).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    ( indep([[Vr,Vrs]]) ->
        multiply(V0s,V1,Vrs) &
        vmul(V0,V1,Vr)
    ;
        multiply(V0s,V1,Vrs),
        vmul(V0,V1,Vr) ).

vmul([],[],0).
vmul([H1|T1],[H2|T2],Vr) :-
    scalar_mult(H1,H2,H1xH2) &
    vmul(T1,T2,T1xT2),
    Vr is H1xH2+T1xT2.

scalar_mult(H1,H2,H1xH2) :- H1xH2 is H1*H2.

... and the parallelized program with this information.
Sharing + Freeness Domain

- Allows detecting failure of groundness checks.
- Increases accuracy of sharing information.
- **Abstract Domain:** \( D_\alpha = D_{\alpha-\text{sharing}} \times D_{\alpha-\text{freeness}} \)
  - \( D_{\alpha-\text{sharing}} = \wp(\wp(P\text{var})) \)
  - \( D_{\alpha-\text{freeness}} = \wp(P\text{var}) \)
- **Abstraction (freeness) of a substitution:**
  \( \alpha_{\text{freeness}}(\theta) = \{ X \mid X \in \text{dom}(\theta), \exists Y \in U\text{var} \ (X \theta = Y) \} \)
- **Example:**
  \( \theta = \{ W/P, X/f(P,Q), Y/g(Q,R), Z/f(a) \} \).
  \( \alpha(\{ \theta \}) = (\lambda_{\text{sharing}}, \lambda_{\text{freeness}}) \), where
  - \( \lambda_{\text{sharing}} = \{ \emptyset, \{ Y \}, \{ W, X \}, \{ X, Y \} \} \)
  - \( \lambda_{\text{freeness}} = \{ W \} \)
The Sharing+Freeness Abstract Domain – A Pictorial Representation

- Two components: sharing & freeness ($\hat{\theta}_{SH}, \hat{\theta}_{FR}$)
- The freeness information restricts the possible combinations of sharing patterns.
- Pictorial representation:

\[
p(X,Y,Z) \\
\hat{\theta}_{SH} = [[XY]] \\
\hat{\theta}_{FR} = [Y] \\
X = f(Y) \\
Z = b
\]

\[
p(X,L) \\
\hat{\theta}_{SH} = [[X][XL]] \\
\hat{\theta}_{FR} = [L] \\
X = [Y|L]
\]

\[
p(X,Y,Z) \\
\hat{\theta}_{SH} = [[XY][Z]] \\
\hat{\theta}_{FR} = [Z] \\
X = f(A) \\
Y = f(A)
\]
&-Prolog compilation: examples - IV

:- entry multiply(g,g,f).

multiply([],_,[]).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    multiply(V0s,V1,Vrs),
    vmul(V0,V1,Vr).

vmul([],[],0).
vmul([H1|T1],[H2|T2],Vr) :-
    scalar_mult(H1,H2,H1xH2),
    vmul(T1,T2,T1xT2),
    Vr is H1xH2+T1xT2.

scalar_mult(H1,H2,H1xH2) :-
    H1xH2 is H1*H2.

Sharing+Freeness information inferred by the analyzer
&-Prolog compilation: examples - IV

multiply([],_,[]).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    multiply(V0s,V1,Vrs) &
    vmul(V0,V1,Vr).

vmul([],[],0).
vmul([H1|T1],[H2|T2],Vr) :-
    scalar_mult(H1,H2,H1xH2) &
    vmul(T1,T2,T1xT2),
    Vr is H1xH2+T1xT2.

scalar_mult(H1,H2,H1xH2) :- H1xH2 is H1*H2.

...and the parallelized program with this information.
## Efficiency of the analyzers — Seconds (’94 numbers!)

<table>
<thead>
<tr>
<th>Program</th>
<th>Average time in seconds</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prol.</td>
<td>S</td>
<td>P</td>
<td>SF</td>
<td>P*S</td>
</tr>
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<td>0.03</td>
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<td>11.52</td>
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<td>2.60</td>
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<td>0.05</td>
<td>0.05</td>
<td>0.08</td>
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<tr>
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<td>16.24</td>
<td>2.26</td>
<td>2.87</td>
</tr>
</tbody>
</table>

- **Prol.** Standard Prolog compiler time
- **S** (Set) Sharing
- **P** Pair sharing (Sondergaard)
- **SF** Sharing + Freeness
- **X*Y** *Combinations*
Dynamic tests ('96 numbers!)

(1-10 processors actual speedups on Sequent Symmetry; 10+ projections using IDRA simulator on execution traces)
A Closer Look at Some Speedups

**Benchmark: mmatrix**

- Simple matrix mul. (> 12 simulated)

**Benchmark: ann**

- The parallelizer, self-parallelized
Independence – Non-Strict Independence

- Pure goals: only one thread “touches” each shared variable. Example:

  ```prolog
  main :- t(X,Y), p(X), q(Y).
  t(X,Y) :- Y = f(X).
  p is independent of t (but p and q are dependent).
  ```

- Impure goals: only rightmost “touches” each shared variable. Example:

  ```prolog
  main :- t(X,Y), p(X), q(Y).
  t(X,Y) :- Y = a. p(X) :- var(X), ..., X=b, ...
  ```

- More parallelism.

- But cannot be detected “a-priori:” requires global analysis.
Independence – Non-Strict Independence

- Very important in programs using “incomplete structures.”

```prolog
flatten(Xs, Ys) :- flatten(Xs, Ys, []).  

flatten([], Xs, Xs).
flatten([X|Xs], Ys, Zs) :- flatten(X, Ys, Ys1), flatten(Xs, Ys1, Zs).
flatten(X, [X|Xs], Xs) :- atomic(X), X \== [].
```

- Another example:

```prolog
qsort([], S, S).
qsort([X|Xs], S, S2) :-
    partition(Xs, X, L, R),
    qsort(L, S, [X|S1]),
    qsort(R, S1, S2).
```
We consider the parallelization of pairs of goals.

Let the situation be: \( \{ \tilde{\beta} \} p \{ \tilde{\psi} \} \ldots q \).

We define:

\[ S(p) = \{ L \in \tilde{\beta}_{SH} \mid L \cap \text{var}(p) \neq \emptyset \} \]
\[ \text{SH} = S(p) \cap S(q) = \{ L \in \tilde{\beta}_{SH} \mid L \cap \text{var}(p) \neq \emptyset \}
\]
\[ \text{and } L \cap \text{var}(q) \neq \emptyset \} \]

- Conditions for non-strict independence for \( p \) and \( q \):
  
  C1 \( \forall L \in \text{SH} \ L \cap \tilde{\psi}_{FR} \neq \emptyset \)
  
  C2 \( \neg (\exists N_1 \ldots N_k \in S(p) \ \exists L \in \tilde{\psi}_{SH}
  \]
  \[ L = \cup_{i=1}^{k} N_i \ \land \ N_1, N_2 \in \text{SH}
  \]
  \[ \land \forall i, j \ 1 \leq i < j \leq k \ N_i \cap N_j \cap \tilde{\beta}_{FR} = \emptyset \}

- C1: preserves freeness of shared variables.
- C2: preserves independence of shared variables.
- More relaxed conditions if information re. partial answers and purity of goals.
Run-Time Checks for NSI Based on ShFr Info

- Run-time checks can be automatically included to ensure NSI when the previous conditions do not hold.
- The method uses analysis information.
- Possible checks are:
  - ground(X): X is ground.
  - allvars(X, F): every free variable in X is in the list F.
  - indep(X, Y): X and Y do not share variables.
  - sharedvars(X, Y, F): every free variable shared by X and Y is in the list F.
- The method generalizes the techniques previously proposed for detection of SI.
- Even when only SI is present, the tests generated may be better than the traditional tests.
Experimental Results

Speedups of five programs that have NSI but no SI:

1. array2list translates an extendible array into a list of index–element pairs.
2. flatten flattens a list of lists of any complexity into a plain list.
3. hanoi_dl solves the towers of Hanoi problem using difference lists.
4. qsort is the sorting algorithm quicksort using difference lists.
5. sparse transforms a binary matrix into an optimized notation for sparse matrices.

<table>
<thead>
<tr>
<th>P</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.78</td>
<td>1.54</td>
<td>2.34</td>
<td>3.09</td>
<td>3.82</td>
<td>4.64</td>
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<td>2.79</td>
<td>3.68</td>
<td>4.50</td>
<td>5.06</td>
<td>5.78</td>
<td>6.75</td>
<td>8.10</td>
<td>8.26</td>
</tr>
</tbody>
</table>
Independence – Constraint Independence

- Standard Herbrand notions do not carry over to general constraint systems.

\[
\text{main} :- Y > X, Z > X, p(Y) \& q(Z), \ldots \]
\[
\text{main} :- Y > X, X > Z, p(Y) \& q(Z), \ldots
\]

- General notion [91-94]: “all constraints posed by second thread are consistent with the output constraints of the first thread.” (Better also for Herbrand!)

- Sufficient a-priori condition: given \( g_1(\bar{x}) \) and \( g_2(\bar{y}) \):

\[
(\bar{x} \cap \bar{y} \subseteq \text{def}(c)) \text{ and } (\exists_{\bar{x}} c \land \exists_{\bar{y}} c \rightarrow \exists_{\bar{y} \cup \bar{x}} c)
\]

(\text{def}(c) \text{ is the set of variables constrained to a unique value in } c)

- For \( c = \{y > x, z > x\} \)
  \( \exists_{\{y\}} c = \exists_{\{z\}} c = \exists_{\{y, z\}} c = \text{true} \)

- For \( c = \{y > x, x > z\} \)
  \( \exists_{\{y\}} c = \exists_{\{z\}} c = \text{true} \), \( \exists_{\{y, z\}} c = y > z \)

- Approximation: presence of “links” through the store.

- Run-time checks: \( \text{def}(X), \text{indep}(X, Y), \text{unlinked}(X, Y) \)
Some Preliminary CLP &-Parallelization Results (Compiler)

- Parallel expressions:

<table>
<thead>
<tr>
<th>Bench. Program</th>
<th>Total CGEs</th>
<th>Uncond. CGEs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Def</td>
<td>Free</td>
</tr>
<tr>
<td>amp</td>
<td>5</td>
<td>–</td>
</tr>
<tr>
<td>bridge</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>circuit</td>
<td>3</td>
<td>2</td>
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<td>dnf</td>
<td>14</td>
<td>14</td>
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<tr>
<td>laplace</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>mining</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>mmatrix</td>
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<td>2</td>
</tr>
<tr>
<td>mg_extend</td>
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<td>0</td>
</tr>
<tr>
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<td>pic</td>
<td>4</td>
<td>3</td>
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<tr>
<td>power</td>
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<td>5</td>
</tr>
<tr>
<td>runge_kutta</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>trapezoid</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Some Preliminary CLP &-Parallelization Results (Compiler)

- Conditional checks:

<table>
<thead>
<tr>
<th>Bench. Program</th>
<th>Conditions: def/unlinked</th>
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<tbody>
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<td></td>
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<tr>
<td>trapezoid</td>
<td>0/9</td>
</tr>
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</table>
Some Preliminary CLP &-Speedup Results (Run-time System)

Speedups for \textit{mmatrix}

\begin{align*}
\text{speedup} & \quad \# \text{processors} \\
1 & \quad 1 \\
2 & \quad 2 \\
3 & \quad 3 \\
4 & \quad 4
\end{align*}

Speedups for critical with \textit{go2} input

\begin{align*}
\text{speedup} & \quad \# \text{processors} \\
1 & \quad 1 \\
2 & \quad 2 \\
3 & \quad 3 \\
4 & \quad 4
\end{align*}

Speedups for critical with \textit{go3} input

\begin{align*}
\text{speedup} & \quad \# \text{processors} \\
1 & \quad 1 \\
2 & \quad 2 \\
3 & \quad 3 \\
4 & \quad 4
\end{align*}
Some Preliminary CLP &-Parallelization Results (Summary)

1. Tests on LP programs:
   - Analysis: compares well to LP-specific domains, but worse relative precision (except Def x Free).
   - Annotation:
     - Efficiency shows the relative precision of the information.
     - Effectiveness comparable for Def x Free. Def and Free alone less precise.

2. Tests on CLP programs:
   - Analysis: acceptable, but comparatively more expensive than for LP.
   - Annotation:
     - Efficiency in the same ratio to analysis as for LP.
     - Effectiveness: Def x Free comparably more effective than Def and Free alone. But still less satisfactory than for LP.
     - Key: none are specific purpose domains.
   - Still, useful speedups.

3. Generalization for LP/CLP with dynamic scheduling and CC [G.Banda Ph.D.].
Other Forms of Independence

- Seen so far:
  - Strict independence / Non-strict independence / Constraint independence
  - Independence in CLP + delay, and non-deterministic CC.
  - Determinacy also a form of independence (e.g., Andorra, AKL, EAM—see later).
    - If/when goals are deterministic they are independent (no-slowdown).
    - If also non-failing then also no speculation (extra work).
      Determinacy actually subsumed by non-strict/search space preserv. definitions!
  - Inconsistency-based independence (“local independence”): finest granularity level, subsumes previous ones.
  - Independence can be applied dynamically and at finer grain levels (e.g., “Local Independence”, DDAS model, AKL stability, etc.).

Some levels of granularity at which independence is applied:
- Goal level / Binding level / Unification level / Across procedures / Etc.
  - “No such thing as dependent and-parallelism.”
Dealing with Speculation

- Computations can be speculative (or even non-terminating!):
  
  \[
  \text{foo}(X) :\leftarrow X=b, \ldots, p(X) \& q(X), \ldots \]
  
  \[
  \text{foo}(X) :\leftarrow X=a, \ldots \]
  
  \[
  p(X) :\leftarrow \ldots, X=a, \ldots \]
  
  \[
  q(X) :\leftarrow \text{large computation}. \]

  but “no slow-down” guaranteed if
  
  ◦ left-biased scheduling,
  
  ◦ instantaneous killing of siblings (failure propagation).

- Left biased schedulers, dynamic throttling of speculative tasks, non-failure, etc. 

- Static detection of non-failure: avoids speculativeness / guarantees theoretical speedup. 
  → importance of non-failure analysis.
Dealing with Overheads, Irregularity

- Independence not enough:
  overheads (task creation and scheduling, communication, etc.)

- In CLP compounded by the fact that the number and size of tasks is highly irregular and dependent on run-time parameters.

- Dynamic solutions:
  - Minimize task management and data communication overheads (micro tasks, shared heaps, compile-time elimination of locks, ...)
  - Efficient dynamic task allocation (e.g., non-centralized task stealing)

- Quite good results for shared-memory multiprocessors early on (e.g., Sequent Balance 1986-89).

- Not sufficient for clusters or over a network.
Dealing with Overheads, Irregularity: Granularity Control

- Replace parallel execution with sequential execution (or vice-versa) based on bounds (or estimations) on task size and overheads.

- Cannot be done completely at compile-time: cost often depends on input (hard to approximate at compile time, even w/abstract interpretation).

```prolog
main :- read(X), read(Z), inc_all(X,Y) & r(Z,M), ...
inc_all([]) := [].
inc_all([I|Is]) := [ I+1 | ~inc_all(Is) ].
```

- Our approach:
  - Derive at compile-time cost functions (to be evaluated at run-time) that efficiently bound task size (lower, upper bounds).
  - Transform programs to carry out run-time granularity control.

![Diagram](image-url)
Granularity Control Example

• For the previous example:

main :- read(X), read(Z), inc_all(X,Y) & r(Z,M), ...
inc_all([]) := [].
inc_all([I|Is]) := [ I+1 | ~inc_all(Is) ].

• Assume X determined to be input, Y output, cost function inferred
  \(2 \times \text{length}(X) + 1\), threshold 100 units:

main :- read(X), read(Z), (2*length(X)+1 > 100 -> inc_all(X,Y) & r(Z,M))
          ; inc_all(X,Y) , r(Z,M)),

• Provably correct techniques (thanks to abstract interpretation):
  can \textit{ensure speedup} if assumptions hold.

• Issues: derivation of data measures, data size functions, task cost functions,
  program transformations, optimizations...
Inference of Bounds on Argument Sizes and Procedure Cost in CiaoPP

1. Perform type/mode inference:
   
   ```prolog
   :- true inc_all(X,Y) : list(X,int), var(Y) => list(Y,int).
   ```

2. Infer size measures: list length.

3. Use data dependency graphs to determine the relative sizes of structures that variables point to at different program points – infer argument size relations:

   \[
   \begin{align*}
   \text{Size}^{2}_{\text{inc\_all}}(0) &= 0 \quad \text{(boundary condition from base case)}, \\
   \text{Size}^{2}_{\text{inc\_all}}(n) &= 1 + \text{Size}^{2}_{\text{inc\_all}}(n - 1).
   \end{align*}
   \]

   Sol = \text{Size}^{2}_{\text{inc\_all}}(n) = n.

4. Use this, set up recurrence equations for the computational cost of procedures:

   \[
   \begin{align*}
   \text{Cost}^{L}_{\text{inc\_all}}(0) &= 1 \quad \text{(boundary condition from base case)}, \\
   \text{Cost}^{L}_{\text{inc\_all}}(n) &= 2 + \text{Cost}^{L}_{\text{inc\_all}}(n - 1).
   \end{align*}
   \]

   Sol = \text{Cost}^{L}_{\text{inc\_all}}(n) = 2n + 1.

   - We obtain lower/upper bounds on task granularities.
   - Non-failure (absence of exceptions) analysis needed for lower bounds.
Granularity Control: Some Refinements/Optimizations (1)

- Simplification of cost functions:
  
  ..., ( length(X) > 50 -> inc_all(X,Y) & r(Z,M) 
  
  ; inc_all(X,Y), r(Z,M) ), ...

  ..., ( length_gt(LX,50) -> inc_all(X,Y) & r(Z,M) 
  
  ; inc_all(X,Y), r(Z,M) ), ...

- Complex thresholds: use also communication cost functions, load, ...

[Example:] Assume $CommCost(inc\_all(X)) = 0.1 (length(X) + length(Y))$. We know $ub\_length(Y)$ (actually, exact size) = $length(X)$; thus:

\[
2 \cdot length(X) + 1 > 0.1 (length(X) + length(X)) \approx \\
2 \cdot length(X) > 0.2 length(X) \equiv \\
2 > 0.2
\]

Guaranteed speedup for any data size!

⇒ Sometimes static decisions can be made despite dynamic sizes and costs (e.g., when ratios are independent of input).
Granularity Control: Some Refinements/Optimizations (1)

- Static task clustering (loop unrolling / data parallelism):
  
  \[
  \ldots, \left( \text{has\_more\_elements\_than}(X, 5) \rightarrow \text{inc\_all\_2}(X, Y) \ & \ r(X) \right) \ ; \ \text{inc\_all\_2}(X, Y), \ r(X) \right), \ldots
  \]

  \[
  \text{inc\_all}([X_1, X_2, X_3, X_4, X_5 | R]) := [X_1 + 1, X_2 + 1, X_3 + 1, X_4 + 1, X_5 + 1 | \sim \text{inc\_all}(R)].
  \]

  \[
  \text{inc\_all}([]) := [].
  \]

  (actually, cases for 4, 3, 2, and 1 elements also have to be included); this is also useful to achieve fast task startup.

- Sometimes static decisions can be made despite dynamic sizes and costs (e.g., when the ratios are independent of input).

- Data size computations can often be done on-the-fly.

- Static placement.
Granularity Control System Output Example

\[
g_qsort([], []).
\]
\[
g_qsort([\text{First}|L1], L2) :-
\]
\[
\text{partition3o4o}(\text{First}, L1, Ls, Lg, \text{Size}_Ls, \text{Size}_Lg),
\]
\[
\text{Size}_Ls > 20 \rightarrow (\text{Size}_Lg > 20 \rightarrow g_qsort(Ls, Ls2) \& g_qsort(Lg, Lg2)
\]
\[
; g_qsort(Ls, Ls2), s_qsort(Lg, Lg2)
\]
\[
; (\text{Size}_Lg > 20 \rightarrow s_qsort(Ls, Ls2), g_qsort(Lg, Lg2)
\]
\[
; s_qsort(Ls, Ls2), s_qsort(Lg, Lg2)))
\]
\[
\text{append}(Ls2, [\text{First}|Lg2], L2).
\]
## Granularity Control: Experimental Results

- **Shared memory:**

<table>
<thead>
<tr>
<th>programs</th>
<th>seq. prog.</th>
<th>no gran.ctl</th>
<th>gran.ctl</th>
<th>gc.stopping</th>
<th>gc.argsize</th>
</tr>
</thead>
<tbody>
<tr>
<td>fib(19)</td>
<td>1.839</td>
<td>0.729</td>
<td>1.169</td>
<td>0.819</td>
<td>0.549</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-60%</td>
<td>-12%</td>
<td>+24%</td>
<td></td>
</tr>
<tr>
<td>hanoi(13)</td>
<td>6.309</td>
<td>2.509</td>
<td>2.829</td>
<td>2.399</td>
<td>2.399</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-12.8%</td>
<td>+4.4%</td>
<td>+4.4%</td>
<td></td>
</tr>
<tr>
<td>unbmatrix</td>
<td>2.099</td>
<td>1.009</td>
<td>1.339</td>
<td>0.870</td>
<td>0.870</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-32.71%</td>
<td>+13.78%</td>
<td>+13.78%</td>
<td></td>
</tr>
<tr>
<td>qsort(1000)</td>
<td>3.670</td>
<td>1.399</td>
<td>1.790</td>
<td>1.659</td>
<td>1.409</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-28%</td>
<td>-19%</td>
<td>-0.0%</td>
<td></td>
</tr>
</tbody>
</table>

- **Cluster:**

<table>
<thead>
<tr>
<th>programs</th>
<th>seq. prog.</th>
<th>no gran.ctl</th>
<th>gran.ctl</th>
<th>gc.stopping</th>
<th>gc.argsize</th>
</tr>
</thead>
<tbody>
<tr>
<td>fib(19)</td>
<td>1.839</td>
<td>0.970</td>
<td>1.389</td>
<td>1.009</td>
<td>0.639</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-43%</td>
<td>-4.0%</td>
<td>+34%</td>
<td></td>
</tr>
<tr>
<td>hanoi(13)</td>
<td>6.309</td>
<td>2.690</td>
<td>2.839</td>
<td>2.419</td>
<td>2.419</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-5.5%</td>
<td>+10.1%</td>
<td>+10.1%</td>
<td></td>
</tr>
<tr>
<td>unbmatrix</td>
<td>2.099</td>
<td>1.039</td>
<td>1.349</td>
<td>0.870</td>
<td>0.870</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-29.84%</td>
<td>+16.27%</td>
<td>+16.27%</td>
<td></td>
</tr>
<tr>
<td>qsort(1000)</td>
<td>3.670</td>
<td>1.819</td>
<td>2.009</td>
<td>1.649</td>
<td>1.429</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-11%</td>
<td>+9.3%</td>
<td>+21%</td>
<td></td>
</tr>
</tbody>
</table>
Refinements (2): Granularity-Aware Annotation

- With classic annotators (MEL, UDG, CDG, ...) we applied granularity control after parallelization:

  

  ![Diagram showing granularity control](image)

  

  - Developed new annotation algorithm that takes task granularity into account:
    - Annotation is a heuristic process (several alternatives possible).
    - Taking task granularity into account during annotation can help make better choices and speed up annotation process.
    - Tasks with larger cost bounds given priority, small ones not parallelized.

  

  ![Diagram showing granularity-driven annotation](image)

  

  \[ \text{Granularity-driven annotation} \quad (\text{assuming} \ g_2 \text{ "small" and} \ g_1 \text{ large if} \ \text{gran}_\text{cond} ) \]
Granularity-Aware Annotation: Concrete Example

- Consider the clause: \[ p :- a, b, c, d, e. \]
- Assume that the dependencies detected between the subgoals of \( p \) are given by:

![Diagram showing dependencies between subgoals a, b, c, d, e]

- Assume also that:

\[ T(a) < T(c) < T(e) < T(b) < T(d), \]

where \( T(i) < T(j) \) means: cost of subgoal \( i \) is smaller than the cost of \( j \).

<table>
<thead>
<tr>
<th>MEL annotator:</th>
<th>( a, b &amp; c, d &amp; e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UDG annotator:</td>
<td>( c &amp; ( a, b, e ), d )</td>
</tr>
<tr>
<td>Granularity-aware:</td>
<td>( a, c, ( b &amp; d ), e )</td>
</tr>
</tbody>
</table>
Refinements (3): Using Execution Time Bounds/Estimates

- Use estimations/bounds on *execution time* for controlling granularity (instead of steps/reductions).
- Execution time generally dependent on platform characteristics ($\approx$ constants) and input data sizes (unknowns).
- Platform-dependent, one-time calibration using fixed set of programs:
  - Obtains value of the platform-dependent constants (costs of basic operations).
- Platform-independent, compile-time analysis:
  - Infers cost functions (using modification of previous method), which return count of *basic operations* given input data sizes.
  - Incorporate the constants from the calibration.
  - We obtain functions yielding *execution times* depending on size of input.
- Predicts execution times with *reasonable* accuracy (challenging!).
- Improving by taking into account lower level factors (current work).
Execution Time Estimation: Concrete Example

- Consider \texttt{nrev} with mode:
  
  \[
  \text{:- pred nrev/2 : list(int) * var.}
  \]

- Estimation of execution time for a concrete input—consider:

  \[A = [1,2,3,4,5], \; \pi = \text{length}(A) = 5\]

<table>
<thead>
<tr>
<th>component</th>
<th>Once (K_{\omega_i})</th>
<th>Static Analysis (\text{Cost}_p(I(\omega_i), \pi) = C_i(\pi))</th>
<th>Application (K_{\omega_i} \times C_i(5))</th>
</tr>
</thead>
<tbody>
<tr>
<td>step</td>
<td>21.27</td>
<td>(0.5 \times n^2 + 1.5 \times n + 1)</td>
<td>21</td>
</tr>
<tr>
<td>nargs</td>
<td>9.96</td>
<td>(1.5 \times n^2 + 3.5 \times n + 2)</td>
<td>57</td>
</tr>
<tr>
<td>giunif</td>
<td>10.30</td>
<td>(0.5 \times n^2 + 3.5 \times n + 1)</td>
<td>31</td>
</tr>
<tr>
<td>gounif</td>
<td>8.23</td>
<td>(0.5 \times n^2 + 0.5 \times n + 1)</td>
<td>16</td>
</tr>
<tr>
<td>viunif</td>
<td>6.46</td>
<td>(1.5 \times n^2 + 1.5 \times n + 1)</td>
<td>45</td>
</tr>
<tr>
<td>vounif</td>
<td>5.69</td>
<td>(n^2 + n)</td>
<td>30</td>
</tr>
</tbody>
</table>

Execution time \(\overline{K}_{\Omega} \bullet \text{Cost}_p(I(\Omega), \pi)\): 1926.8
Fib 15, 1 processor
Fib 15, 8 processors (same scale)

(VisAndOr output.)
Fib 15, 8 processors (full scale)

(VisAndOr output.)
Fib 15, 8 processors, with granularity control (same scale)

(VisAndOr output.)
**Dependent And–parallelism: DDAS (I)**

- Exploits Independent + “Dependent” And–parallelism.
- Goals communicate through shared variables.
- Shared variables are marked (\texttt{dep/1} annotation).
- Example:
  \[
  \text{example}(X) :- (\text{dep}(X) \Rightarrow a(X) \& b(X)).
  \]
  \[
  a(X). \quad b(1).
  \]
- To retain sequential search space: dependent variables are bound by only one producer and received by some consumers.
  - The producer can bind the variable.
  - A consumer suspends if it tries to bind the variable.
  - A suspended consumer is resumed if the variable on which it is suspended is bound \textit{or if it becomes leftmost}.
  - Producer for a given variable changes dynamically as goals finish execution:
    “The producer for a dependent variable is the (lexicographically) leftmost \textbf{active} task which has access to that variable.”
Dependent And–parallelism: DDAS (II)

- Performance:
  - IAP speedups + new dependent-and speedups
  - IAP programs with one agent run at about 50% speed w.r.t. sequential execution (due to locking and other overheads).
  - DAP programs run at 30%–40% lower speed.
Andorra

- Basic Andorra model [D.H.D. Warren]: goals for which at most one clause matches should be executed first (inspired by Naish’s PNU-Prolog).
- If a solution exists, computation rule is complete and correct for pure programs (switching lemma). (But otherwise finite failures can become infinite failures.)
- Determinate reductions can proceed in parallel without the need of choice points \(\rightarrow\) no dependent backtracking needed.
  - Prolog support: preprocessor + engine (interpreter).
  - Exploits both and- and or-parallelism. (Good speedups in practice)
  - Problem: no nondeterministic steps can proceed in parallel.
  - With implicit control (unspecified) [Warren, Gupta]
  - With explicit/implicit control: AKL [Janson, Haridi ILPS91] (implicit rule – “stability”: non-deterministic steps can proceed if “they cannot affected” by other steps)
Non-restricted And-Parallelism

• Classical parallelism operator 
  &/2: nested fork-join.

• However, more flexible constructions can be used to denote (non-restricted) and-parallelism:

  ◊ G &> H
  — schedules goal G for parallel execution and continues executing the code after G &> H.

  * H_G is a *handler* which contains / points to the state of goal G.

  ◊ H_G <&
  — waits for the goal associated with H_G to finish.

  * The goal H_G was associated to has produced a solution; bindings for the output variables are available.

• Optimized deterministic versions: &!/2, <&!/1.

• Operator &/2 can be written as:

  A & B :- A &> H, call(B), H <&.
Non-restricted And-Parallelism

- More parallelism can be exploited with these primitives.
- Take the sequential code below (dep. graph to the right) and three possible parallelizations:

\[
p(X,Y,Z) :-
\begin{align*}
  & a(X,Z), \\
  & b(X), \\
  & c(Y), \\
  & d(Y,Z).
\end{align*}
\]

\[
p(X,Y,Z) :-
\begin{align*}
  & a(X,Z) & c(Y), \\
  & b(X) & d(Y,Z).
\end{align*}
\]

\[
p(X,Y,Z) :-
\begin{align*}
  & c(Y) & (a(X,Z),b(X)), \\
  & d(Y,Z).
\end{align*}
\]

Sequential  Restricted IAP  Unrestricted IAP

- In this case: unrestricted parallelization at least as good (time-wise) as any restricted one, assuming no overhead.
Annotation algorithms for non-restricted \&-par.: general idea

- Main idea:
  - *Publish goals* (e.g., $G \&> H$) as soon as possible.
  - *Wait for results* (e.g., $H \&<$) as late as possible.
  - One clause at a time.

- Limits to how soon a goal is published + how late results are gathered are given by the dependencies with the rest of the goals in the clause.

- As with $\&/2$, annotation may respect or not relative order of goals in clause body.
  - Order determined by $\&>/2$.
  - Order not respected $\Rightarrow$ more flexibility in annotation.
### Performance Results – Speedups

<table>
<thead>
<tr>
<th>Benchm.</th>
<th>Ann.</th>
<th>Number of processors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>AIAKL</td>
<td>UMEL</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>UOUDG</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>UDG</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>UUDG</td>
<td>0.97</td>
</tr>
<tr>
<td>Hanoi</td>
<td>UMEL</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>UOUDG</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>UDG</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>UUDG</td>
<td>0.89</td>
</tr>
<tr>
<td>FibFun</td>
<td>UMEL</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>UOUDG</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>UDG</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>UUDG</td>
<td>0.99</td>
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<tr>
<td>Takeuchi</td>
<td>UMEL</td>
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<td></td>
<td>UOUDG</td>
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<td></td>
<td>UDG</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>UUDG</td>
<td>0.88</td>
</tr>
</tbody>
</table>
Performance results - Restricted vs. Unrestricted And-Parallelism

AIAKL

Hanoi

FibFun

Sun Fire T2000 - 8 cores

Takeuchi
Towards a higher-level implementation

- Versions of and-parallelism previously implemented: 
  &-Prolog, &-ACE, AKL, Andorra-I,...
  rely on complex low-level machinery. Each agent:

- Our objective: alternative, easier to maintain implementation approach.

- Fundamental idea: raise non-critical components to the source language level:
  - Prolog-level: goal publishing, goal searching, goal scheduling, “marker” creation (through choice-points),...
  - C-level: low-level threading, locking, untrailing,...
  → Simpler machinery and more flexibility.
  → Easily exploits unrestricted IAP.

- Current implementation (for shared-memory multiprocessors):
  - Each agent: sequential Prolog machine + goal list + (mostly) Prolog code.

- Recently added full parallel backtracking!
(Preliminary) performance results Sun Fire T2000 - 8 cores

**Boyer-Moore**

- Boyer-Moore
- Boyer-Moore with granularity control

**Fibonacci**

- Fibonacci
- Fibonacci with granularity control

**Quicksort**

- Quicksort
- Quicksort with difference lists
- Quicksort with granularity control

**Takeuchi**

- Takeuchi, Restricted version
- Takeuchi, Unrestricted version
And–parallel Execution Models: Summary (I)

- Different types of parallelism, with different costs associated:
  - Complexity considerations (search space, speculation).
  - Coordination cost for agreeing on unifiable bindings.
- Overheads / granularity control.
- Approaches:
  - IAP: goals do not restrict each other’s search space.
    * Ensures no slow-down w.r.t. sequential execution.
    * Retains as much as possible WAM optimizations.
    * Some parallelism lost.
- NSIAP: IAP + . . .
  - At most one goal can bind to non-variable a shared variable (or they make compatible bindings) and no goal aliases shared variables.
  - Generalization: search space preservation.
  - Reduced to IAP via program analysis and transformation.
And–parallel Execution Models: Summary (II)

- DDAS: goals communicate bindings.
  * Incorporate a suspension mechanism to ensure no more work than in a sequential system – “fine grained independence”.
  * Handle dependent backtracking.
  * Some locking and variable-management overhead.

- Andorra I: determinate depend. and– + or–parallelism
  * Dependent determinate goals run in parallel.
  * Allows incorporating also or–parallelism easily.
  * Some locking and goal-management overhead.

- Extended Andorra Model – adding independent and parallelism to Andorra-I.
  * With implicit control.
  * With explicit control: AKL.
Other developments

- **ACE**: combining MUSE and &-Prolog (And/or Copy-based Execution model) [Being developed by New Mexico S.U. and UPM]
  - ngc-recomputation dep-compiler

- Interesting work on memory management [Pontelli ICLP’95].

- Visualization Tools (VisiPAL, ViMust, VisAndOr, Vista, etc.)
  - [?, ?, ?]

- Fine-grained compile-time parallelization (“local indep” [Bueno et al 1994])

- Distributed systems:
  - Significant progress made (e.g. UCM work [Araujo et al] and Ciao).
  - Vital component: granularity control.

- **Ciao**: Concurrent Constraint Independent And/Or-Parallel System [‘92-present]
  - Non-deterministic concurrent constraint language.
  - Subsumes Prolog, CLP, CC (+Andorra via transformation), ...
Distributed / net execution.

- Most Prolog systems have a notion of threads nowadays (SICStus, Ciao, SWI, Yap, XSB, B-Prolog, ), adequate for hand-coding coarse-grain parallelism.
Some comparison with work in other paradigms

- Much progress (e.g., in FORTRAN) for regular computations. But comparatively less on:
  - parallelization across procedure calls,
  - irregular computations,
  - complex data structures / pointers,
  - speculation, etc.
Wrap-up: (C)LP strong points

- Several generations of parallelizing compilers for LP and CLP [85-...]:
  - Good compilation speed, proved correct and efficient.
  - Speedups over state-of-the-art sequential systems on applications.
  - Good demonstrators of abstract interpretation as data-flow analysis technique.
  - Now including granularity control.

Improved on hand parallelizations on several large applications.

- Areas of particularly good progress:
  - Concepts of independence (pointers, search/speculation, constraints...).
  - Inter-procedural analysis (dynamic data, recursion, pointers/aliasing, etc.).
  - Parallelization algorithms for conditional dependency graphs.
  - Dealing with irregularity:
    * efficient task representation and fast dynamic scheduling,
    * static inference of task cost functions – granularity control.
  - Mixed static/dynamic parallelization techniques.
Wrap-up: areas for improvement

- Weaker areas / shortcomings:
  - In general, weak in detecting independence in structure traversals based on integer arithmetic (modeled as recursions over recursive data structures to fit parallelizer).
  - Weaker partitioning / placement for regular computations and static data structures.
  - Little work on mutating data structures (e.g., single assignment transformations).

- The objective is to perform *all* these tasks well also!

- Opportunities for synergy.

- A final plug for constraint programming:
  - Merges elegantly the symbolic and the numerical worlds.
  - We believe many of the features of CLP will make it slowly into mainstream languages (e.g., ILOG, ALMA, and other recent proposals).
Some general-purpose contributions from (C)LP

- Some examples so far:
  - Stealing-based scheduling strategies and microthreading.
  - Cactus-like stack memory management techniques.
  - Abstract interpretation-based static dependency analysis.
  - Sharing (aliasing) analyses, Shape analyses, ...
  - Parallelization (“annotation”) algorithms.
  - Cost analysis-based granularity control.
  - Logic variable-based synchronization.
  - Determinacy-based parallelization.
  - ...
Some challenges?

- Parallelism not yet exploited on an everyday basis (real system, real applications).
- Some challenges:
  - Scalability of techniques (from analysis to scheduling).
  - Maintainability of the systems: simplification?
    * Move as much as possible to source level?
      (And explore this same route with many other things – e.g., tabling)
  - Better automatic parallelization:
    * Better granularity control (e.g., time-based).
    * Better granularity-aware annotators.
    * Full scalability of analysis (modular analysis, etc.).
    * Automate program transformations (e.g., loop unrollings).
  - Supporting multiple types of parallelism easily is still a challenge.
  - A really elegant (and implementable) concurrent language which includes non-determinism.
  - Combination w/low-level optimization and other features (r.g., or-// YapTab).
Some Bibliography (for a general tutorial see [GPA+01])


[Cas08] A. Casas. *Automatic Unrestricted Independent And-Parallelism in Declarative Multiparadigm Languages*. PhD thesis, University of New Mexico (UNM), Electrical and Computer Engineering Department, University of New Mexico, Albuquerque, NM 87131-0001 (USA), September 2008.


