Parallel Execution of Logic Programs
A Tutorial
(Or: Multicores are here! Now, what do we do with them?)

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Introduction / Motivation

- Multicore chips have moved parallelism from niche (HPC) to mainstream –even on laptops!

- According to vendors (and Intel in particular [e.g., DAMP workshops]):
  - Feature size reductions will continue for foreseeable future (12 generations!).
  - But power consumption does not allow increasing clock speeds much.
  - Multicore is the way to use this space without raising power consumption.
  - Number of cores expected to double with each generation!

- But writing parallel programs hard/error-prone –how to exploit all those cores?
  - Ideal situation: Conventional Program + Multiprocessor = Higher Perf.
    → automatic parallelization.
  - More realistically: compiler-aided parallelization.
  - Languages (dialects, constructs) for parallelization+parallel programming.
  - Scheduling techniques [BW93, Cie92], memory management, abstract machines, etc.
LP and CLP – quite interesting from the parallelism point of view

- Many parallelism-friendly aspects:
  - program close to problem description → less hiding of intrinsic parallelism
  - well understood mathematical foundation → simplifies formal treatment
  - relative purity (well behaved variable scoping, fewer side-effects, generally single assignment) → more amenable to automatic parallelization.

- At the same time, requires dealing with the most complex problems \cite{Her97, Her00}:
  - irregular computations; complex data structures; (well behaved) pointers; dynamic memory management; recursion; ...

  *but in a much more elegant context;*
  and brings up some upcoming issues (e.g., speculation, search, constraints).

  → Very good platform for developing universally useful techniques:
  Examples to date: conditional dep. graphs, abstract interpretation w/interesting domains, cost analysis / gran. control, dynamic sched. and load balancing, ...
Complex Data Structures / Pointers

- Example:

```
main :- X = f(Y,Z),
Y = a,
W = Z,
W = g(K),
X = f(a,g(b)).
```
Parallelism in Logic Programs and CLP

- **Or-parallelism**[^Con83]: execute simultaneously different search space branches.
  - Present in general search problems, enumeration part of constr. problems, etc.

```prolog
money(S,E,N,D,M,O,R,Y) :- digit(0).
digit(S), digit(1).
digit(E), ... 
..., digit(9).
carry(I), 
..., carry(0).
N is E+O-10*I, carry(1).
```

- **And-parallelism**[^Con83]: execute simultaneously different clause body goals.
  - Comprises traditional parallelism (parallel loops, divide and conquer, etc.).
  - *Concurrent languages* also generally based on and-parallelism.

```prolog
qsort([X|L],R) :-
  partition(L,X,L1,L2),
  qsort(L2,R2),
  qsort(L1,R1),
  append(R1,[X|R2],R).
```

[^Con83]: Con83
Objective and Issues

- Temptation: make use of all this potential.
- Problem: this can yield a slowdown or even erroneous results.
- Objective [HR89]: and/or-parallel execution of (some of the goals in) logic programs (and full Prolog, CLP, CC, ...), while:
  - obtaining the same solutions as the sequential execution (i.e., correctness)
  - taking a shorter or equal execution time (speedup or, at least, no-slowdown over state-of-the-art sequential systems) (i.e., efficiency).

- Above conditions may not always be met:
  - Independence: conditions that the run-time behavior of the goals must satisfy to guarantee correctness and efficiency (under ideal conditions – no overhead).

- The presence of overheads complicates things further:
  - Granularity Control: techniques for ensuring efficiency in the presence of overheads.
Sequential and Parallel Execution Framework: OR

- Model [HR95]: consider a state \( G = \langle g_1 : g_2 : \ldots : g_n, \theta \rangle \) where we select \( g_1 \).

- If there are two clauses:
  
  \[
  g'_1 \leftarrow g'_{11}, \ldots, g'_{1m}.
  \]
  
  \[
  g_1 \leftarrow g''_{11}, \ldots, g''_{1k}.
  \]

- We construct two states:
  
  \[
  G' = \langle g'_{11} : \ldots : g'_{1m} : g_2 : \ldots : g_n, \theta' \theta' \rangle
  \]
  
  \[
  G'' = \langle g''_{11} : \ldots : g''_{1k} : g_2 : \ldots : g_n, \theta' \theta'' \rangle
  \]

- **Sequential execution**: execute \( G' \) first and then \( G'' \).

- **Parallel execution**: execute \( G' \) and \( G'' \) in parallel.

- Since \( G' \) and \( G'' \) are completely independent [HR95]:
  
  - ◦ Same results are obtained in parallel or sequentially.
  - ◦ All branches can be explored in parallel.
  - ◦ Same number of branches explored (only if “all sols”!).

- Thus, or-parallelism: mostly implementation issues.
  
  (but side-effects, cuts, and aggregation predicates complicate things)
Issues in OR Parallelism

- System organization:
  - System comprises a collection of workers (processes/processors).
  - Each worker is an LP/CLP engine with a full set of stacks.
  - A scheduler assigns unexplored branches to idle workers.

- Main implementation problem: alternative bindings – efficiently maintaining different environments per branch (e.g., $p_1$ and $p_2$ in example):
  - Sharing (e.g. Aurora [LBD+88], PEPSys/ECLIPSE [CSW88 ECR93], etc.)
  - Recomputation (e.g. Delphi model) [Clo87].
  - Copying (e.g. Muse system) [AK90] ECLIPSE [ECR93], SICStus, OZ).
  - Theoretical limitations [GJ93]. Desirable:
    - Constant–time access to variables
    - Constant–time task creation
    - Constant–time task switching

Impossible to meet all three with a finite number of processors. (Hence, they are not met in sequential execution!)
Issues in Or-parallelism: Illustration

..., p(X), ...

\[ p_1(X) :\ldots, \text{X=a, }\ldots, !, \ldots \]
\[ p_2(X) :\ldots, \text{X=b, }\ldots \]

main :- l, s.

:- parallel l/0.
\[ l :\text{ large\_work\_a.} \]
\[ l :\text{ large\_work\_b.} \]

:- parallel s/0.
\[ s :\text{ small\_work\_a.} \]
\[ s :\text{ small\_work\_b.} \]
Issues in OR Parallelism

- **Speculation** (e.g., \( p_2 \) in example).
  - To guarantee **speedup**: avoid speculative work – too strong/difficult?
  - To guarantee **no-slowdown**:
    * Left-biased scheduling.
    * Instantaneous killing on cut.

- **Granularity**: avoid parallelizing work that is too small.

- Parallelization can be done:
  - Adding `parallel/1` annotations to selected predicates (ANL,ECLIPSE)
  - Others (Aurora, MUSE) automatically via the scheduler.

- Useful supporting techniques identified:
  - Visualization/trace analysis: ANL, VisAndOr/IDRA [CGH93, FCH96], ViMust, Parsee [PK96], VisAll [FIVC98], ...
  - Program transformation to increase granularity [Pre93].
  - Compile-time/run-time granularity control; automatically introduce `parallel` annotations [LGDH96].
Some Results in OR Parallelism

- Quite successful systems built (ECLIPSE, SICSTUS/MUSE, Aurora, OrpYap\textsuperscript{RSS99}, etc.)

- MUSE is quite easy to add to an existing Prolog system (done with Prolog by BIM, also added to SICStus Prolog V3.0)

- Significant speedups w.r.t. state-of-the-art Prolog systems can be obtained with Aurora and Muse \textit{for search-based applications}.

<table>
<thead>
<tr>
<th>Program</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>10</th>
<th>Sicstus 0.6</th>
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<tbody>
<tr>
<td>parse1</td>
<td>1</td>
<td>1.8</td>
<td>2.8</td>
<td>2.93</td>
<td>2.76</td>
<td>1.25</td>
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<tr>
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<td>1.97</td>
<td>3.74</td>
<td>6.92</td>
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<td>1</td>
<td>2.07</td>
<td>4.06</td>
<td>7.81</td>
<td>9.59</td>
<td>1.43</td>
</tr>
</tbody>
</table>

- Much work done on schedulers (left bias, cut, side effects, ...)

- Easy to extend to CLP (e.g., VanHentenryck\textsuperscript{Van89}, ECLIPSE system).
Simple Goal-level And-Parallel Exec. Framework

- **Model [HR90][HR95]:**
  consider a state $G = \langle g_1 : g_2 : \ldots : g_n, \theta \rangle$, to execute $g_1$ and $g_2$ in parallel:
  - execute $\langle g_1, \theta \rangle$ and $\langle g_2, \theta \rangle$ in parallel (fork) obtaining $\theta_1$ and $\theta_2$,
  - continue with $\langle g_3 : \ldots : g_n, \theta_1 \theta_2 \rangle$ (join).

- **Regarding multiple solutions – two possibilities:**
  - Gather all solutions for both goals separately.
  - Perform “parallel backtracking”.

- **Multiple problems**, related to *variable binding conflicts*: during parallel execution of $\langle g_1, \theta \rangle$ and $\langle g_2, \theta \rangle$ the same variable may be bound to inconsistent values.

- **Correctness problems** (due to the definition of composition of substitutions – e.g. $x/a$ composed with $x/b$ succeeds!) [HR89]
  Solutions (proved correct in case of “pure” goals):
  - Modify definition of composition: $\theta \circ \eta(t) = \text{mgu}(E(\theta) \cup E(\eta))(t)$
  - Change parallel model.
  - Not an issue in CLP: conjunction instead of composition [GHM93][GHM00].
Issues in And-Parallelism – Independence

- **Correctness**: “same” solutions as sequential execution.
- **Efficiency**: execution time < than seq. program (or, at least, no-slowdown: ≤).
  (We assume parallel execution has no overhead in this first stage.)

<table>
<thead>
<tr>
<th></th>
<th>Imperative</th>
<th>Functions</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>$Y := W+2$</td>
<td>(+ $W$ 2)</td>
<td>$Y = W+2,$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>$X := Y+Z$</td>
<td>(+ $Z$ )</td>
<td>$X = Y+Z,$</td>
</tr>
<tr>
<td>read-write deps</td>
<td>strictness</td>
<td></td>
<td>cost!</td>
</tr>
</tbody>
</table>

- Running at $s_2$ “seeing $s_1$”:

  For *Predicates* (multiple procedure definitions):

  ```
  main :-
      $s_1$ p(X),
      $s_2$ q(X),
      write(X).
  
  p(X) :- X=a.
  q(X) :- X=b, *large computation.*
  q(X) :- X=a.
  ```

  Again, cost issue: *if* $p$ *affects* $q$ (*prunes its choices*) *then* $q$ *ahead of* $p$ *is speculative.*

- **Independence**: condition that guarantees correctness *and efficiency.*
Independence and its Detection

- Informal notion: a computation “does not affect” another (also referred to as “stability” in, e.g., EAM/AKL).

- Greatly clarified when put in terms of Search Space Preservation (SSP) – shown SSP sufficient and necessary condition for efficiency \([\text{GHM93, Gar94}]\).

- Detection of independence:
  - Run-time (a-priori conditions) \([\text{Con83, LK88, JH91}]\).
  - Compile-time \([\text{CDD85}]\).
  - Mixed: conditional execution graph expressions \([\text{DeG84, Her86b}]\). (1)
  - User control: explicit parallelism (concurrent languages). (2)

- \((1)+(2) = &\text{-Prolog} \([\text{DeG84, Her86b}]\): view parallelization as a source to source transformation of original program into a parallelized (“annotated”) one in a concurrent/parallel language. Allows:
  - Automatic parallelization — and understanding the result).
  - User parallelization — and the compiler checking it).
Concrete System Used in Examples: Ciao

• For concreteness, hereafter we use &-Prolog (now Ciao) as a target. The relevant minimal subset of &-Prolog/Ciao:
  ◦ Prolog (with if-then-else, etc.).
  ◦ Parallel conjunction “&/2”
    (with correct and complete forwards and backwards semantics).
  ◦ A number of primitives for run-time testing of instantiation state.

• Ciao \[HC94, HBC^+99, HBC^+08, BCC^+09\] is one of the popular Prolog/CLP systems (supports ISO-Prolog fully).
  Many other features: new-generation multi-paradigm language/prog.env. with:
  ◦ Predicates, constraints, functions (including lazyness), higher-order, ...
  ◦ Assertion language for expressing rich program properties
    (types, shapes, pointer aliasing, non-failure, determinacy, data sizes, cost, ...).
    Static debugging, verification, program certification, PCC, ...
  ◦ Parallel, concurrent, and distributed execution primitives.
    * Automatic parallelization.
    * Automatic granularity and resource control.
A Priori Independence: Strict Independence-I

- Approach (goal level). Consider parallelizing \( p(X,Y) \) and \( q(X,Z) \):

  ```prolog
  main :-
    t(X,Y,Z),
    s1 p(X,Y),
    s2 q(X,Z).
  ```

  We compare the behaviour of \( s2 q(X,Z) \) and \( s1 q(X,Z) \).

- **A-priori Independence**: when reasoning only about \( s1 \).
  Can be checked at run-time before execution of the goals.

- **A priori independence in the Herbrand domain**: Strict Independence [DeG84, HR89]: goals do not share variables at run-time.

- **Example 1**: Above, if \( t(X,Y,Z) :- X=a \).
The “pointers” view:

correctness and efficiency (search space preservation) guaranteed for \( p \) & \( q \) if there are no “pointers” between \( p \) and \( q \).

\[
\text{main :- } X=f(K,g(K)), \ Y=a, \\
\text{ } Z=g(L), \ W=h(b,L), \\
\text{ } ---------------------> \\
\text{ } p(X,Y), \\
\text{ } q(Y,Z), \\
\text{ } r(W).
\]

\( p \) and \( q \) are strictly independent, but \( q \) and \( r \) are not.
A Priori Independence: Strict Independence-III

• Example 2:

\[
qs([X|L],R) :- \text{part}(L,X,L1,L2),
    qs(L2,R2), qs(L1,R1),
    \text{app}(R1,[X|R2],R).
\]

Might be annotated in &-Prolog (or Ciao) as:

\[
qs([X|L],R) :-
    \text{part}(L,X,L1,L2),
    ( \text{indep}(L1,L2) -> qs(L2,R2) \& qs(L1,R1) \n    \text{; qs}(L2,R2), qs(L1,R1) ),
    \text{app}(R1,[X|R2],R).
\]

• Not always possible to determine locally/statically:

\[
\text{main} :- t(X,Y), p(X), q(Y).
\]

\[
\text{main} :- \text{read}([X,Y]), p(X), q(Y).
\]

• Alternatives: run-time independence tests, global analysis, ...
Fundamental issues:

- Can we build a system which obtains speedups w.r.t. a state of the art sequential LP system using such annotations?

- Can those annotations be generated automatically?
And-Parallelism Implementation

- By translation to or-parallelism [ECR93 CDO88]:
  - Simplicity
  - Relatively high overhead $\rightarrow$ higher need for granularity control
  - Used, e.g., in ECLIPSE system.

- Direct implementation [Her86b]:
  - Abstract machine needs to be modified: e.g., PWAM / Marker model [Her87 Her86a SH96 PG98], EAM/AKL box machine [War90 JH90].
    * System comprises a collection of agents (processes/processors).
    * Each agent is an LP/CLP engine with a full set of stacks.
    * Scheduling is normally done lazily through goal stacks.
  - Low overhead (but granularity control still useful)
  - Direct support for concurrent/parallel language
  - Used in &-Prolog/Ciao and most other systems: ACE, IDIOM, DDAS, ...

- Also, higher-level implementations (see later).
And-Parallelism Implementation

- Issues in direct implementation:
  - Scheduling / fast task startup.
  - Memory management.
  - Use of analysis information to improve indexing.
  - Local environment support.
  - Recomputation vs. copying.
  - Efficient implementation of parallel backtracking (and opportunities for intelligent backtracking).
  - Efficient implementation of “ask” (for communication among threads).
  - etc.
&-Prolog Run-time System: PWAM architecture

- Evolution of the RAP-WAM (the first Multisequential Model?) and Sicstus WAM.

- Defined as a storage model + an instruction set

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&-Prolog Run-time System: Agents and Stack Sets

- Agents separate from Stack Sets; Dynamic creation/deletion of S.Sets/Agents
- Lazy, on demand scheduling

- Extensions / optimizations:
  - DASWAM / DDAS System (dependent and-//) [She92, She96]
  - &ACE, ACE Systems (or-, and-, dep-//) [PG95a, GHPSC94a, PGPF97]
Sequent Symmetry, hand parallelized programs.
(Speedup over state of the art sequential systems.)
Visualization of And-parallelism – (small) qsort, 1 processor

(VisAndOr CGH93 output.)
Visualization of And-parallelism – (small) qsort, 4 processors

(VisAndOr CGH93 output.)
• Not always possible to determine locally/statically:

```
main :- t(X,Y), p(X), q(Y).
```

```
main :- read([X,Y]), p(X), q(Y).
```

• Alternatives: run-time independence tests, global analysis, ...

```
main :- read([X,Y]), ( indep(X,Y) -> p(X) & q(Y) ; p(X), q(Y) ).
```

```
main :- t(X,Y), p(X) & q(Y).  %% (After analysis)
```
Parallelization Process: CDG-based Automatic Parallelization

- **Conditional Dependency Graph** (of some code segment) \[\text{HW87, BGH99, GPA}^{+01}\):
  - Vertices: possible tasks (statements, calls, bindings, etc.).
  - Edges: possible dependencies (labels: conditions needed for independence).

- Local or global analysis used to reduce/remove checks in the edges.
- Annotation process converts graph back to parallel expressions in source.

\[
\text{foo}(\ldots) : - \\
\quad g_1(\ldots), \\
\quad g_2(\ldots), \\
\quad g_3(\ldots).
\]
Simplifying Independence Conditions (Strict Ind.)

- Recall that $b_1$ and $b_2$ are strictly independent for $\theta$ iff

$$\text{vars}(b_1\theta) \cap \text{vars}(b_2\theta) = \emptyset$$

- $\text{indep}(b_1, b_2)$ iff $b_1$ and $b_2$ do not share variables at run–time.

- $p(x, y)$ and $q(y, z)$ are strictly independent at run–time iff $\text{indep}\{x, y\}, \{y, z\}$.

- Equivalent to $\{\text{indep}(x, y), \text{indep}(x, z), \text{indep}(y, y), \text{indep}(y, z)\}$.

- Domain of interpretation $DI$: subset of propositional logic.

- For clause $C$, it contains predicates of the form $\text{ground}(x)$ and $\text{indep}(y, z)$, $\{x, y, z\} \subseteq \text{vars}(C)$, with axioms:

$$\{\text{ground}(x) \rightarrow \text{indep}(x, y)\} \{x, y\} \subseteq \text{vars}(C)\}$$

$$\{\text{indep}(x, x) \rightarrow \text{ground}(x)\} x \in \text{vars}(C)\}$$

- The set $\{\text{indep}(x, y), \text{indep}(x, z), \text{indep}(y, y), \text{indep}(y, z)\}$ can be simplified to $\{\text{ground}(y), \text{indep}(x, z)\}$. 
Identify Dependencies

\[ p(x,y) \rightarrow q(x,z) \rightarrow s(z,w) \]

\[ \text{ind}(y,w) \rightarrow \text{falsetrue} \]

Analysis

Info

Simplify Dependencies

\[ h(x,y,z) : - (p(x,y) \& q(x,z)), s(z,w). \]

\[ h(x,y,z) : - \text{ind}(y,w) \rightarrow p(x,y) \& (q(x,z), s(z,w)) ; p(x,y), q(x,z), s(z,w). \]
Annotators (local dependency analysis)
MEL/CDG/UDG/URLP/...
Abstract Interpretation (Sharing, Sharing+Freeness, Aeqs, Def, Lsign, ...)
Dependency Info
side−effect analysis
granularity analysis
Parallelized Code (&)
Ciao/&−Prolog
Parallel RT system
Ciao: (C)LP, FP, (Java) ...
PARALLELIZING COMPILER (CiaoPP)
Parallelizing compiler [HW87] (now integrated in CiaoPP [HBPLG99, HPBLG03]):

- **Global Analysis**: infers independence information.
- **Annotator(s)**: Prolog $\rightarrow$ &-Prolog parallelization [DeG87, MH90, BGH94a, CH94, PGPF97, MBdlBH99].
  - MEL: Maximum Expression Length — simple heuristic.
  - CDG: Conditional Graph Expressions — graph partitioning of clauses.
  - UDG: Unconditional Graph Expressions.
  - Variants of CDG/UDG.
  - Enhanced to better use global analysis info and granularity information (still on-going).

- **Low-level PWAM compiler**: extension of Sicstus V0.5
- **Granularity Analysis**: determines task size or size functions [DLH90, DL91, DL93, DLGHL94, DLGHL97, DLGH97, SCK98, MLGCH08].
- **Granularity Control**: restricts parallelism based on task sizes [DLH90, LGHD96, SCK98].
- **Other modules**: side effect analyzer (sequencing of side-effects, coded in &-Prolog), multiple specializer / partial evaluator, invariant eliminator, etc.
multiply([],_,[]).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    vmul(V0,V1,Vr),
    multiply(V0s,V1,Vrs).

vmul([],[],0).
mmul([H1|T1],[H2|T2],Vr) :-
    scalar_mult(H1,H2,H1xH2),
    vmul(T1,T2,T1xT2),
    Vr is H1xH2+T1xT2.

scalar_mult(H1,H2,H1xH2) :- H1xH2 is H1*H2.
\begin{verbatim}
multiply([],_,[]).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    (  ground([V1]), indep([[V0,V0s],[V0,Vrs],[V0s,Vr],[Vr,Vrs]])
        -> vmul(V0,V1,Vr) & multiply(V0s,V1,Vrs)
      ;  vmul(V0,V1,Vr), multiply(V0s,V1,Vrs)  ).

vmul([],[],0).
vmul([H1|T1],[H2|T2],Vr) :-
    (  indep([[H1,T1],[H1,T2],[T1,H2],[H2,T2]])
        -> scalar_mult(H1,H2,H1xH2) & vmul(T1,T2,T1xT2)
      ;  scalar_mult(H1,H2,H1xH2), vmul(T1,T2,T1xT2)  ),
    Vr is H1xH2+T1xT2.

scalar_mult(H1,H2,H1xH2) :- H1xH2 is H1*H2.
\end{verbatim}

Parallelized program (&-Prolog/Ciao)—no global analysis
Dependency Analysis: Global Analysis Subsystem

- “PLAI” analyzer – top-down driven bottom up analysis (enhanced version of Bruynooghe’s scheme).
- Optimized fixpoint algorithm (keeps track of dependencies and approximation state of information, avoids recomputation).
- Some useful abstract domains:
  - Sharing Domain Abstraction (“S”).
  - Sharing+Freeness Domain Abstraction (“SF”).
  - Sondergaard’s ASub (linearity) domain (“P”).
  - Type domains, depth-K, etc.
  - (Constraints:) Definiteness, Freeness, LSign domains.
- Domains combined using framework: e.g. ASub+SH, ASub+ShF.
- Automatic elimination of repetitive checks.
- Current analyzer quite robust, with support for a relatively complete set of builtins.
- Support for full Prolog, CLP(R), etc.
“Sharing” Abstraction (Groundness + Set Sharing)

- **Definitions:**
  - $Uvar$: universe of all variables,
  - $Pvar$: set of program variables in a clause,
  - $Subst$: set of all possible mappings from variables in $Pvar$ to terms.

- **Abstract Domain:** $D_\alpha = \wp(\wp(Pvar))$

- **Abstraction of a substitution:**
  $\alpha(A) : Subst \rightarrow D_\alpha$
  $\alpha(\theta) = \{\text{Occ}(\theta, U) | U \in Uvar\}$ where $\text{Occ}(\theta, U) = \{X | X \in \text{dom}(\theta) \land U \in \text{var}(X \theta)\}$,

- **Example:** Let $\theta = \{W = a, X = f(A_1, A_2), Y = g(A_2), Z = A_3\}$.
  $\alpha(\theta) = \{\emptyset, \{X\}, \{X, Y\}, \{Z\}\}$.

- **Note that**
  - $independent(x\theta, y\theta) \iff \nexists v \in Uvar, x \in \text{Occ}(\theta, v) \land y \in \text{Occ}(\theta, v)$

  Other additional axioms are encoded in the sharing patterns.
:- entry multiply(g,g,f).

multiply([],_,[]).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    multiply(V0s,V1,Vrs),
    vmul(V0,V1,Vr).

vmul([],[],0).
vmul([H1|T1],[H2|T2],Vr) :-
    scalar_mult(H1,H2,H1xH2),
    vmul(T1,T2,T1xT2),
    Vr is H1xH2+T1xT2.

scalar_mult(H1,H2,H1xH2) :-
    H1xH2 is H1*H2.

Sharing information inferred by the analyzer
multiply([],_,[]).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    ( indep([[Vr,Vrs]]) ->
      multiply(V0s,V1,Vrs) &
      vmul(V0,V1,Vr)
    ;
      multiply(V0s,V1,Vrs),
      vmul(V0,V1,Vr) ).

vmul([],[],0).
vmul([H1|T1],[H2|T2],Vr) :-
    scalar_mult(H1,H2,H1xH2) &
    vmul(T1,T2,T1xT2),
    Vr is H1xH2+T1xT2.

scalar_mult(H1,H2,H1xH2) :- H1xH2 is H1*H2.

...and the parallelized program with this information.
Sharing + Freeness Domain

- Allows detecting failure of groundness checks.
- Increases accuracy of sharing information.

**Abstract Domain:** $D_\alpha = D_{\alpha \text{-sharing}} \times D_{\alpha \text{-freeness}}$

\begin{align*}
\diamond D_{\alpha \text{-sharing}} &= \varnothing(\varnothing(P\text{var})) \\
\diamond D_{\alpha \text{-freeness}} &= \varnothing(P\text{var})
\end{align*}

- Abstraction (freeness) of a substitution:
  
  $\alpha_{\text{freeness}}(\theta) = \{ X \mid X \in \text{dom}(\theta), \exists Y \in U\text{var} (X\theta = Y)\}$

- Example:

  $\theta = \{W/P, X/f(P, Q), Y/g(Q, R), Z/f(a)\}$.

  $\alpha(\{\theta\}) = (\lambda_{\text{sharing}}, \lambda_{\text{freeness}})$, where

  \begin{align*}
  \diamond \lambda_{\text{sharing}} &= \{\emptyset, \{Y\}, \{W, X\}, \{X, Y\}\} \\
  \diamond \lambda_{\text{freeness}} &= \{W\}$
Two components: sharing & freeness ($\hat{\theta}_{SH}, \hat{\theta}_{FR}$)

The freeness information restricts the possible combinations of sharing patterns.

Pictorial representation:

\[
\begin{align*}
p(X,Y,Z)  \\
\hat{\theta}_{SH} &= [[XY]] \\
\hat{\theta}_{FR} &= [Y] \\
X &= f(Y) \\
Z &= b
\end{align*}
\]

\[
\begin{align*}
p(X,L)  \\
\hat{\theta}_{SH} &= [[X][XL]] \\
\hat{\theta}_{FR} &= [L] \\
X &= [Y|L]
\end{align*}
\]

\[
\begin{align*}
p(X,Y,Z)  \\
\hat{\theta}_{SH} &= [[XY][Z]] \\
\hat{\theta}_{FR} &= [Z] \\
X &= f(A) \\
Y &= f(A)
\end{align*}
\]
:- entry multiply(g,g,f).

multiply([],_,[]).  
multiply([V0|V0s],V1,[Vr|Vrs]) :-  
    multiply(V0s,V1,Vrs),  
    vmul(V0,V1,Vr).%

vmul([],[],0).  
vmul([H1|T1],[H2|T2],Vr) :-  
    scalar_mult(H1,H2,H1xH2),  
    vmul(T1,T2,T1xT2),  
    Vr is H1xH2 + T1xT2.%

scalar_mult(H1,H2,H1xH2) :-  
    H1xH2 is H1*H2.

Sharing + Freeness information inferred by the analyzer
multiply([],_,[]).
multiply([V0|V0s],V1,[Vr|Vrs]) :-
    multiply(V0s,V1,Vrs) &
    vmul(V0,V1,Vr).

vmul([],[],0).
vmul([H1|T1],[H2|T2],Vr) :-
    scalar_mult(H1,H2,H1xH2) &
    vmul(T1,T2,T1xT2),
    Vr is H1xH2+T1xT2.

scalar_mult(H1,H2,H1xH2) :- H1xH2 is H1*H2.

...and the parallelized program with this information.
### Efficiency of the analyzers — Seconds ('94 numbers!)

<table>
<thead>
<tr>
<th>Program</th>
<th>Average time in seconds</th>
<th>Prol.</th>
<th>S</th>
<th>P</th>
<th>SF</th>
<th>P*S</th>
<th>P*SF</th>
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<td>0.09</td>
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<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
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<td>2.87</td>
<td>3.42</td>
</tr>
</tbody>
</table>

Prol. Standard Prolog compiler time

S (Set) Sharing

P Pair sharing (Sondergaard)

SF Sharing + Freeness

X*Y Combinations

[BGH94b] [MBdlBH99] [BGH99]
Dynamic tests (’96 numbers!)

(1-10 processors actual speedups on Sequent Symmetry; 10+ projections using IDRA simulator on execution traces) [BGH94b, MBdIBH99, BGH99]
A Closer Look at Some Speedups

**Benchmark: mmatrix**

- **Simple matrix mul. (> 12 simulated)**
- **The parallelizer, self-parallelized**

M. Hermenegildo – Parallel Execution of Logic Programs

Compulog/ALP Summer School – Las Cruces, NM, July 24-27 2008
Independence – Non-Strict Independence

• Pure goals: only one thread “touches” each shared variable. Example:

\[
\text{main} :- t(X,Y), p(X), q(Y).
\]

\[
t(X,Y) :- Y = f(X).
\]

\(p\) is independent of \(t\) (but \(p\) and \(q\) are dependent).

• Impure goals: only rightmost “touches” each shared variable. Example:

\[
\text{main} :- t(X,Y), p(X), q(Y).
\]

\[
t(X,Y) :- Y = a. \quad p(X) :- \text{var}(X), \ldots, X=b, \ldots
\]

• More parallelism.

• But cannot be detected “a-priori:” requires global analysis.
Independence – Non-Strict Independence

• Very important in programs using “incomplete structures.”

\[
\text{flatten}(Xs, Ys) :- \text{flatten}(Xs, Ys, []). \\
\text{flatten}([], Xs, Xs). \\
\text{flatten}([X|Xs], Ys, Zs) :- \text{flatten}(X, Ys, Ys1), \text{flatten}(Xs, Ys1, Zs). \\
\text{flatten}(X, [X|Xs], Xs) :- \text{atomic}(X), X \neq [].
\]

• Another example:

\[
\text{qsort}([], S, S). \\
\text{qsort}([X|Xs], S, S2) :- \\
\quad \text{partition}(Xs, X, L, R), \\
\quad \text{qsort}(L, S, [X|S1]), \\
\quad \text{qsort}(R, S1, S2).
\]
We consider the parallelization of pairs of goals.

Let the situation be: \( \{ \beta \} p \{ \psi \} \ldots q \).

We define:

\[
S(p) = \{ L \in \beta_{SH} \mid L \cap \text{var}(p) \neq \emptyset \}
\]

\[
SH = S(p) \cap S(q) = \{ L \in \beta_{SH} \mid L \cap \text{var}(p) \neq \emptyset \wedge L \cap \text{var}(q) \neq \emptyset \}
\]

Conditions for non-strict independence for \( p \) and \( q \):

\[
\begin{align*}
C1 \quad & \forall L \in SH \ L \cap \tilde{\psi}_{FR} \neq \emptyset \\
C2 \quad & \neg (\exists N_1 \ldots N_k \in S(p) \ \exists L \in \tilde{\psi}_{SH} \\
& \quad L = \bigcup_{i=1}^{k} N_i \wedge N_1, N_2 \in SH \\
& \quad \wedge \forall i, j \ 1 \leq i < j \leq k \ N_i \cap N_j \cap \beta_{FR} = \emptyset
\end{align*}
\]

- C1: preserves freeness of shared variables.
- C2: preserves independence of shared variables.
- More relaxed conditions if information re. partial answers and purity of goals.
Run-Time Checks for NSI Based on ShFr Info

- Run-time checks can be automatically included to ensure NSI when the previous conditions do not hold.
- The method uses analysis information.
- Possible checks are:
  - ground(X): X is ground.
  - allvars(X,F): every free variable in X is in the list F.
  - indep(X,Y): X and Y do not share variables.
  - sharedvars(X,Y,F): every free variable shared by X and Y is in the list F.
- The method generalizes the techniques previously proposed for detection of SI.
- Even when only SI is present, the tests generated may be better than the traditional tests.
Experimental Results

Speedups of five programs that have NSI but no SI:

1. **array2list** translates an extendible array into a list of index–element pairs.
2. **flatten** flattens a list of lists of any complexity into a plain list.
3. **hanoi_dl** solves the towers of Hanoi problem using difference lists.
4. **qsort** is the sorting algorithm quicksort using difference lists.
5. **sparse** transforms a binary matrix into an optimized notation for sparse matrices.

<table>
<thead>
<tr>
<th>P</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>4.84</td>
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<td>5.06</td>
<td>5.78</td>
<td>6.75</td>
<td>8.10</td>
<td>8.26</td>
</tr>
</tbody>
</table>
Independence – Constraint Independence

- Standard Herbrand notions do not carry over to general constraint systems.

\[
\text{main} :- Y > X, Z > X, p(Y) \& q(Z), \ldots \\
\text{main} :- Y > X, X > Z, p(Y) \& q(Z), \ldots
\]

- General notion [91-94]: “all constraints posed by second thread are consistent with the output constraints of the first thread.” (Better also for Herbrand!)

- Sufficient a-priori condition: given \(g_1(\bar{x})\) and \(g_2(\bar{y})\):

\[
(\bar{x} \cap \bar{y} \subseteq \text{def}(c)) \text{ and } (\exists \neg \bar{x}c \wedge \exists \neg \bar{y}c \rightarrow \exists \neg \bar{y} \cup \bar{x}c)
\]

\(\text{def}(c)\) is the set of variables constrained to a unique value in \(c\)

- For \(c = \{y > x, z > x\}\)
  \[\exists \neg\{y\}c = \exists \neg\{z\}c = \exists \neg\{y, z\}c = \text{true}\]

- For \(c = \{y > x, x > z\}\)
  \[\exists \neg\{y\}c = \exists \neg\{z\}c = \text{true}, \quad \exists \{y, z\}c = y > z\]

- Approximation: presence of “links” through the store.

- Run-time checks: \(\text{def}(X), \text{indep}(X, Y), \text{unlinked}(X, Y)\)
Some Preliminary CLP &-Parallelization Results (Compiler)

GBH96

- Parallel expressions:

<table>
<thead>
<tr>
<th>Bench. Program</th>
<th>Total CGEs</th>
<th>Uncond. CGEs</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Def</td>
<td>Free</td>
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<tr>
<td>amp</td>
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<td>–</td>
</tr>
<tr>
<td>bridge</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>circuit</td>
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<td>14</td>
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<tr>
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<td>–</td>
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<td>4</td>
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<tr>
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<td>pic</td>
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<tr>
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<td>1</td>
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<tr>
<td>trapezoid</td>
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</tbody>
</table>
Some Preliminary CLP &-Parallelization Results (Compiler)

- Conditional checks:

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Some Preliminary CLP &-Speedup Results (Run-time System)

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<td>4</td>
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Speedups for matrix

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<th>4</th>
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Speedups for critical with go2 input

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<td>3</td>
<td>4</td>
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Speedups for critical with go3 input
Some Preliminary CLP &-Parallelization Results (Summary)

1. Tests on LP programs:
   - Analysis: compares well to LP-specific domains, but worse relative precision (except Def x Free).
   - Annotation:
     ◦ Efficiency shows the relative precision of the information.
     ◦ Effectiveness comparable for Def x Free. Def and Free alone less precise.

2. Tests on CLP programs:
   - Analysis: acceptable, but comparatively more expensive than for LP.
   - Annotation:
     ◦ Efficiency in the same ratio to analysis as for LP.
     ◦ Effectiveness: Def x Free comparably more effective that Def and Free alone. But still less satisfactory than for LP.
     ◦ Key: none are specific purpose domains.
   - Still, useful speedups.

3. Generalization for LP/CLP with dynamic scheduling and CC [G.Banda Ph.D.].
Other Forms of Independence

- Seen so far:
  - Strict independence / Non-strict independence / Constraint independence
  - Independence in CLP + delay \[GHM96\], and non-deter. CC \[BHMR94, BHMR98\].
  - Determinacy also a form of independence (e.g., Andorra, AKL, EAM –see later).
    - If/when goals are deterministic they are independent (no-slowdown).
    - If also non-failing then also no speculation (extra work).
  Determinacy actually subsumed by non-strict/search space preserv. definitions!
  - Inconsistency-based independence (“local independence”): finest granularity level, subsumes previous ones \[BHMR94, BHMR98\].
  - Independence can be applied dynamically and at finer grain levels (e.g., “Local Independence”, DDAS model, AKL stability, etc.) \[HC94\].

Some levels of granularity at which independence is applied:

- Goal level / Binding level / Unification level / Across procedures / Etc.

→ “No such thing as dependent and-parallelism.”
Dealing with Speculation

- Computations can be speculative (or even non-terminating!):
  
  ```
  foo(X) :- X=b, ..., p(X) & q(X), ...
  foo(X) :- X=a, ...
  
  p(X) :- ..., X=a, ...
  
  q(X) :- large computation.
  ```

  but “no slow-down” guaranteed if
  - left-biased scheduling,
  - instantaneous killing of siblings (failure propagation).

- Left biased schedulers, dynamic throttling of speculative tasks, non-failure, etc. [HR89, HR95, Gar94].

- Static detection of non-failure [BCMH94, DLGH97]:
  avoids speculativeness / guarantees theoretical speedup.
  → importance of non-failure analysis.
Dealing with Overheads, Irregularity

- Independence not enough: overheads (task creation and scheduling, communication, etc.)
- In CLP compounded by the fact that the number and size of tasks is highly irregular and dependent on run-time parameters.
- Dynamic solutions:
  - Minimize task management and data communication overheads (micro tasks, shared heaps, compile-time elimination of locks, ...)
  - Efficient dynamic task allocation (e.g., non-centralized task stealing)
- Quite good results for shared-memory multiprocessors early on (e.g., Sequent Balance 1986-89).
- Not sufficient for clusters or over a network.
Dealing with Overheads, Irregularity: Granularity Control

- Replace parallel execution with sequential execution (or vice-versa) based on bounds (or estimations) on task size and overheads.

- Cannot be done completely at compile-time: cost often depends on input (hard to approximate at compile time, even w/abstract interpretation).

\[
\text{main :- read(X), read(Z), inc_all(X,Y) \& r(Z,M), ...}
\]
\[
\text{inc_all([]) := [].}
\]
\[
\text{inc_all([I|Is]) := [ I+1 | \sim \text{inc_all(Is) } ].}
\]

- Our approach:
  - Derive at compile-time cost functions (to be evaluated at run-time) that efficiently bound task size (lower, upper bounds).
  - Transform programs to carry out run-time granularity control.
Granularity Control Example

- For the previous example:

  ```prolog
  main :- read(X), read(Z), inc_all(X,Y) & r(Z,M), ...
  inc_all([]) := [].
  inc_all([I|Is]) := [ I+1 | ~inc_all(Is) ].
  ```

- Assume X determined to be input, Y output, cost function inferred $2 \times \text{length}(X) + 1$, threshold 100 units:

  ```prolog
  main :- read(X), read(Z), (2*length(X)+1 > 100 -> inc_all(X,Y) & r(Z,M) ; inc_all(X,Y) , r(Z,M)), ...
  ```

- Provably correct techniques (thanks to abstract interpretation): can ensure speedup if assumptions hold.

- Issues: derivation of data measures, data size functions, task cost functions, program transformations, optimizations...

n
Inference of Bounds on Argument Sizes and Procedure Cost in CiaoPP

1. Perform type/mode inference:
   \[ \texttt{:- true inc\_all(X,Y) : list(X,int), var(Y) \Rightarrow list(Y,int)}. \]

2. Infer size measures: list length.

3. Use data dependency graphs to determine the relative sizes of structures that
   variables point to at different program points – infer argument size relations:

   \[
   \begin{align*}
   \text{Size}_{\text{inc\_all}}(0) &= 0 \quad \text{(boundary condition from base case)}, \\
   \text{Size}_{\text{inc\_all}}(n) &= 1 + \text{Size}_{\text{inc\_all}}(n - 1). \\
   \end{align*}
   \]

   \[ \text{Sol} = \text{Size}_{\text{inc\_all}}(n) = n. \]

4. Use this, set up recurrence equations for the computational cost of procedures:

   \[
   \begin{align*}
   \text{Cost}_{\text{inc\_all}}^L(0) &= 1 \quad \text{(boundary condition from base case)}, \\
   \text{Cost}_{\text{inc\_all}}^L(n) &= 2 + \text{Cost}_{\text{inc\_all}}^L(n - 1). \\
   \end{align*}
   \]

   \[ \text{Sol} = \text{Cost}_{\text{inc\_all}}^L(n) = 2n + 1. \]

- We obtain lower/upper bounds on task granularities.
- Non-failure (absence of exceptions) analysis needed for lower bounds.
Granularity Control: Some Refinements/Optimizations (1)

- Simplification of cost functions:
  
  ..., ( length(X) > 50 \rightarrow \text{inc\_all}(X,Y) \land r(Z,M) \\
  \quad ; \text{inc\_all}(X,Y), r(Z,M) ), ... 

  
  ..., ( length\_gt(LX,50) \rightarrow \text{inc\_all}(X,Y) \land r(Z,M) \\
  \quad ; \text{inc\_all}(X,Y), r(Z,M) ), ... 

- Complex thresholds: use also communication cost functions, load, ...

  **Example:** Assume \( \text{CommCost}(\text{inc\_all}(X)) = 0.1 \ (\text{length}(X) + \text{length}(Y)) \). We know \( \text{ub}\_\text{length}(Y) \) (actually, exact size) = \( \text{length}(X) \); thus:

  \[ 2 \ \text{length}(X) + 1 > 0.1 \ (\text{length}(X) + \text{length}(X)) \equiv \]

  \[ 2 \ \text{length}(X) > 0.2 \ \text{length}(X) \equiv \]

  Guaranteed speedup for any data size! \( \Leftarrow \) \( \text{2} > 0.2 \)

  \( \Rightarrow \) Sometimes static decisions can be made despite dynamic sizes and costs (e.g., when ratios are independent of input).
Granularity Control: Some Refinements/Optimizations (1)

- Static task clustering (loop unrolling / data parallelism):

  ..., ( has_more_elements_than(X,5) -> inc_all_2(X,Y) & r(X)
        ; inc_all_2(X,Y), r(X) ), ...

  inc_all([X1,X2,X3,X4,X5|R) := [X1+1,X2+1,X3+1,X4+1,X5+1] ~inc_all(R).
  inc_all([]) := [].

  (actually, cases for 4, 3, 2, and 1 elements also have to be included); this is also useful to achieve fast task startup [BB93, DJ94, HC95, HC96, GHPSC94b, PG95b].

- Sometimes static decisions can be made despite dynamic sizes and costs (e.g., when the ratios are independent of input).
- Data size computations can often be done on-the-fly.
- Static placement.
Granularity Control System Output Example

\[ g\_qsort([], []). \]
\[ g\_qsort([First|L1], L2) :- \]
\[ \text{partition3o4o(First, L1, Ls, Lg, Size\_Ls, Size\_Lg),} \]
\[ \text{Size\_Ls} > 20 \rightarrow \text{(Size\_Lg} > 20 \rightarrow g\_qsort(Ls, Ls2) \land g\_qsort(Lg, Lg2) \]
\[ \quad \quad ; g\_qsort(Ls, Ls2), s\_qsort(Lg, Lg2)) \]
\[ ; (\text{Size\_Lg} > 20 \rightarrow s\_qsort(Ls, Ls2), g\_qsort(Lg, Lg2) \]
\[ \quad \quad ; s\_qsort(Ls, Ls2), s\_qsort(Lg, Lg2))) \],
\[ \text{append(Ls2, [First|Lg2], L2).} \]

\[ \text{partition3o4o(F, [], [], [], 0, 0).} \]
\[ \text{partition3o4o(F, [X|Y], [X|Y1], Y2, SL, SG) :-} \]
\[ \quad X =\leq F, \text{partition3o4o(F, Y, Y1, Y2, SL1, SG), SL is SL1 + 1.} \]
\[ \text{partition3o4o(F, [X|Y], Y1, [X|Y2], SL, SG) :-} \]
\[ \quad X > F, \text{partition3o4o(F, Y, Y1, Y2, SL, SG1), SG is SG1 + 1.} \]
Granularity Control: Experimental Results

- **Shared memory:**

<table>
<thead>
<tr>
<th>programs</th>
<th>seq. prog.</th>
<th>no gran.ctl</th>
<th>gran.ctl</th>
<th>gc.stopping</th>
<th>gc.argsize</th>
</tr>
</thead>
<tbody>
<tr>
<td>fib(19)</td>
<td>1.839</td>
<td>0.729</td>
<td>1.169</td>
<td>0.819</td>
<td>0.549</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-60%</td>
<td>-12%</td>
<td>+24%</td>
</tr>
<tr>
<td>hanoi(13)</td>
<td>6.309</td>
<td>2.509</td>
<td>2.829</td>
<td>2.399</td>
<td>2.399</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-12.8%</td>
<td>+4.4%</td>
<td>+4.4%</td>
</tr>
<tr>
<td>unbmatrix</td>
<td>2.099</td>
<td>1.009</td>
<td>1.339</td>
<td>0.870</td>
<td>0.870</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-32.71%</td>
<td>+13.78%</td>
<td>+13.78%</td>
</tr>
<tr>
<td>qsort(1000)</td>
<td>3.670</td>
<td>1.399</td>
<td>1.790</td>
<td>1.659</td>
<td>1.409</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-28%</td>
<td>-19%</td>
<td>-0.0%</td>
</tr>
</tbody>
</table>

- **Cluster:**

<table>
<thead>
<tr>
<th>programs</th>
<th>seq. prog.</th>
<th>no gran.ctl</th>
<th>gran.ctl</th>
<th>gc.stopping</th>
<th>gc.argsize</th>
</tr>
</thead>
<tbody>
<tr>
<td>fib(19)</td>
<td>1.839</td>
<td>0.970</td>
<td>1.389</td>
<td>1.009</td>
<td>0.639</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-43%</td>
<td>-4.0%</td>
<td>+34%</td>
</tr>
<tr>
<td>hanoi(13)</td>
<td>6.309</td>
<td>2.690</td>
<td>2.839</td>
<td>2.419</td>
<td>2.419</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-5.5%</td>
<td>+10.1%</td>
<td>+10.1%</td>
</tr>
<tr>
<td>unbmatrix</td>
<td>2.099</td>
<td>1.039</td>
<td>1.349</td>
<td>0.870</td>
<td>0.870</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-29.84%</td>
<td>+16.27%</td>
<td>+16.27%</td>
</tr>
<tr>
<td>qsort(1000)</td>
<td>3.670</td>
<td>1.819</td>
<td>2.009</td>
<td>1.649</td>
<td>1.429</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-11%</td>
<td>+9.3%</td>
<td>+21%</td>
</tr>
</tbody>
</table>
Refinements (2): Granularity-Aware Annotation

- With classic annotators (MEL, UDG, CDG, ...) we applied granularity control after parallelization:

- Developed new annotation algorithm that takes task granularity into account:
  - Annotation is a heuristic process (several alternatives possible).
  - Taking task granularity into account during annotation can help make better choices and speed up annotation process.
  - Tasks with larger cost bounds given priority, small ones not parallelized.
Granularity-Aware Annotation: Concrete Example

- Consider the clause: \( p : - a, b, c, d, e. \)
- Assume that the dependencies detected between the subgoals of \( p \) are given by:

```
\begin{align*}
& a & b \\
& c & d & e
\end{align*}
```

- Assume also that:

\[
T(a) < T(c) < T(e) < T(b) < T(d),
\]

where \( T(i) < T(j) \) means: cost of subgoal \( i \) is smaller than the cost of \( j \).

<table>
<thead>
<tr>
<th>MEL annotator:</th>
<th>( a, b &amp; c, d &amp; e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UDG annotator:</td>
<td>( c &amp; ( a, b, e ), d )</td>
</tr>
<tr>
<td>Granularity-aware:</td>
<td>( a, c, ( b &amp; d ), e )</td>
</tr>
</tbody>
</table>
Refinements (3): Using Execution Time Bounds/Estimates

- Use estimations/bounds on execution time for controlling granularity (instead of steps/reductions).
- Execution time generally dependent on platform characteristics ($\approx$ constants) and input data sizes (unknowns).
- Platform-dependent, one-time calibration using fixed set of programs:
  - Obtains value of the platform-dependent constants (costs of basic operations).
- Platform-independent, compile-time analysis:
  - Infers cost functions (using modification of previous method), which return count of basic operations given input data sizes.
  - Incorporate the constants from the calibration.

→ we obtain functions yielding execution times depending on size of input.
- Predicts execution times with reasonable accuracy (challenging!).
- Improving by taking into account lower level factors (current work).
Execution Time Estimation: Concrete Example

• Consider \( n\text{rev} \) with mode:
  
  \[\text{:- pred nrev/2 : list(int) * var.}\]

• Estimation of execution time for a concrete input —consider:

\[A = [1,2,3,4,5], \; n = \text{length}(A) = 5\]

<table>
<thead>
<tr>
<th>component</th>
<th>Once ( K_{\omega_i} )</th>
<th>Static Analysis ( \text{Cost}_p(I(\omega_i), n) = C_i(n) )</th>
<th>Application ( C_i(5) \times K_{\omega_i} \times C_i(5) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>step</td>
<td>21.27</td>
<td>(0.5 \times n^2 + 1.5 \times n + 1)</td>
<td>21 \times 446.7</td>
</tr>
<tr>
<td>nargs</td>
<td>9.96</td>
<td>(1.5 \times n^2 + 3.5 \times n + 2)</td>
<td>57 \times 567.7</td>
</tr>
<tr>
<td>giunif</td>
<td>10.30</td>
<td>(0.5 \times n^2 + 3.5 \times n + 1)</td>
<td>31 \times 319.3</td>
</tr>
<tr>
<td>gounif</td>
<td>8.23</td>
<td>(0.5 \times n^2 + 0.5 \times n + 1)</td>
<td>16 \times 131.7</td>
</tr>
<tr>
<td>viunif</td>
<td>6.46</td>
<td>(1.5 \times n^2 + 1.5 \times n + 1)</td>
<td>45 \times 290.7</td>
</tr>
<tr>
<td>vounif</td>
<td>5.69</td>
<td>(n^2 + n)</td>
<td>30 \times 170.7</td>
</tr>
</tbody>
</table>

Execution time \( \overline{K_\Omega} \times \text{Cost}_p(I(\Omega), n) \): 1926.8
Fib 15, 1 processor

(\text{VisAndOr CGH93} \text{ output.})
Fib 15, 8 processors (same scale)

(VisAndOr CGH93 output.)
Fib 15, 8 processors (full scale)
Fib 15, 8 processors, with granularity control (same scale)

(VisAndOr CGH93 output.)
Dependent And–parallelism: DDAS (I)

- Exploits Independent + “Dependent” And–parallelism.
- Goals communicate through shared variables.
- Shared variables are marked (dep/1 annotation).
- Example:
  \[
  \text{example}(X) :- (\text{dep}(X) \Rightarrow a(X) \& b(X)).
  \]
  \[
  a(X). \quad b(1).
  \]
- To retain sequential search space: dependent variables are bound by only one producer and received by some consumers.
  - The producer can bind the variable.
  - A consumer suspends if it tries to bind the variable.
  - A suspended consumer is resumed if the variable on which it is suspended is bound or if it becomes leftmost.
  - Producer for a given variable changes dynamically as goals finish execution:
    “The producer for a dependent variable is the (lexicographically) leftmost active task which has access to that variable.”

[She92] [She96]
Dependent And–parallelism: DDAS (II)

- Performance:
  - IAP speedups + new dependent-and speedups
  - IAP programs with one agent run at about 50% speed w.r.t. sequential execution (due to locking and other overheads).
  - DAP programs run at 30%–40% lower speed.
Andorra

- Basic Andorra model [D.H.D.Warren]: goals for which at most one clause matches should be executed first (inspired by Naish’s PNU-Prolog).

- If a solution exists, computation rule is complete and correct for pure programs (switching lemma). (But otherwise finite failures can become infinite failures.)

- Determinate reductions can proceed in parallel without the need of choice points → no dependent backtracking needed.

  - Prolog support: preprocessor + engine (interpreter).
  - Exploits both and- and or-parallelism. (Good speedups in practice)
  - Problem: no nondeterministic steps can proceed in parallel.

  - With implicit control (unspecified) [Warren, Gupta]
  - With explicit/implicit control: AKL [Janson, Haridi ILPS91]
    (implicit rule – “stability”: non-deterministic steps can proceed if “they cannot affected” by other steps)
Non-restricted And-Parallelism

- Classical parallelism operator &/2: nested fork-join.

- However, more flexible constructions can be used to denote (non-restricted) and-parallelism:
  
  $G \&> H$ — schedules goal $G$ for parallel execution and continues executing the code after $G \&> H$.
  
  * $H_G$ is a *handler* which contains / points to the state of goal $G$.

  $H_G <&$ — waits for the goal associated with $H_G$ to finish.
  
  * The goal $H_G$ was associated to has produced a solution; bindings for the output variables are available.

- Optimized deterministic versions: &!/2, <&!/1.

- Operator &/2 can be written as:

  $$A \& B :- A \&> H, \text{call}(B), H <&.$$
Non-restricted And-Parallelism

- More parallelism can be exploited with these primitives.
- Take the sequential code below (dep. graph to the right) and three possible parallelizations:

```
p(X,Y,Z) :-
    a(X,Z),
b(X),
c(Y),
d(Y,Z).
p(X,Y,Z) :-
    a(X,Z) & c(Y),
b(X) & d(Y,Z).
p(X,Y,Z) :-
    a(X,Z),
b(X) &> Hb,
c(Y) &> Hc,
d(Y,Z),
Hb <&,
Hc <&.
```

Sequential | Restricted IAP | Unrestricted IAP

- In this case: unrestricted parallelization at least as good (time-wise) as any restricted one, assuming no overhead.
Annotation algorithms for non-restricted &-par.: general idea

[CCH07]

- Main idea:
  - Publish goals (e.g., \( G \ \&> \ H \)) as soon as possible.
  - Wait for results (e.g., \( H \ <& \)) as late as possible.
  - One clause at a time.

- Limits to how soon a goal is published + how late results are gathered are given by the dependencies with the rest of the goals in the clause.

- As with \&/2, annotation may respect or not relative order of goals in clause body.
  - Order determined by \&>/2.
  - Order not respected \( \Rightarrow \) more flexibility in annotation.
## Performance Results – Speedups

### Benchm. | Ann. | Number of processors |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>AIAKL</td>
<td>UMEL</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>UOUDG</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>UDG</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>UUDG</td>
<td>0.97</td>
</tr>
<tr>
<td>Hanoi</td>
<td>UMEL</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>UOUDG</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>UDG</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>UUDG</td>
<td>0.89</td>
</tr>
<tr>
<td>FibFun</td>
<td>UMEL</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>UOUDG</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>UDG</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>UUDG</td>
<td>0.99</td>
</tr>
<tr>
<td>Takeuchi</td>
<td>UMEL</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>UOUDG</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>UDG</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>UUDG</td>
<td>0.88</td>
</tr>
</tbody>
</table>
Performance results - Restricted vs. Unrestricted And-Parallelism

AIAKL

Hanoi

FibFun

Sun Fire T2000 - 8 cores

Takeuchi
Towards a higher-level implementation

- Versions of and-parallelism previously implemented: &-Prolog, &-ACE, AKL, Andorra-l,...
  rely on complex low-level machinery. Each agent:

- Our objective: alternative, easier to maintain implementation approach.

- Fundamental idea: raise non-critical components to the source language level:
  - **Prolog-level**: goal publishing, goal searching, goal scheduling, “marker” creation (through choice-points),...
  - **C-level**: low-level threading, locking, untrailing,...

→ Simpler machinery and more flexibility.
→ Easily exploits unrestricted IAP.

- Current implementation (for shared-memory multiprocessors):
  - Each agent: sequential Prolog machine + goal list + (mostly) Prolog code.

- Recently added full parallel backtracking!

\[CCH08b\] [CCH08a]
(Preliminary) performance results Sun Fire T2000 - 8 cores

- Boyer-Moore
- Boyer-Moore with granularity control

- Fibonacci
- Fibonacci with granularity control

- Quicksort
- Quicksort with difference lists
- Quicksort with granularity control

- Takeuchi, Restricted version
- Takeuchi, Unrestricted version

M. Hermenegildo – Parallel Execution of Logic Programs

Compulog/ALP Summer School – Las Cruces, NM, July 24-27 2008
And–parallel Execution Models: Summary (I)

- Different types of parallelism, with different costs associated:
  - Complexity considerations (search space, speculation).
  - Coordination cost for agreeing on unifiable bindings.
- Overheads / granularity control.
- Approaches:
  - IAP: goals do not restrict each other’s search space.
    * Ensures no slow-down w.r.t. sequential execution.
    * Retains as much as possible WAM optimizations.
    * Some parallelism lost.
  - NSIAP: IAP + . . .
    - At most one goal can bind to non-variable a shared variable (or they make compatible bindings) and no goal aliases shared variables.
    - Generalization: search space preservation.
    - Reduced to IAP via program analysis and transformation.
DDAS: goals communicate bindings.
* Incorporate a suspension mechanism to ensure no more work than in a sequential system – “fine grained independence”.
* Handle dependent backtracking.
* Some locking and variable-management overhead.

Andorra I: determinate depend. and– + or–parallelism
* Dependent determinate goals run in parallel.
* Allows incorporating also or–parallelism easily.
* Some locking and goal-management overhead.

Extended Andorra Model – adding independent and parallelism to Andorra-I.
* With implicit control.
* With explicit control: AKL.
Other developments

- **ACE**: combining MUSE and &-Prolog (And/or Copy-based Execution model)
  [Being developed by New Mexico S.U. and UPM]
  ngc-recomputation dep-compiler

- Interesting work on memory management [Pontelli ICLP’95].

- Visualization Tools (VisiPAL, ViMust, VisAndOr, Vista, etc.)
  [HN90, CGH93, VPG97, FIVC98, Tic92]

- Fine-grained compile-time parallelization (“local indep” [Bueno et al 1994])

- Distributed systems:
  - Significant progress made (e.g. UCM work [Araujo et. al] and Ciao).
  - Vital component: granularity control.

- **Ciao**: Concurrent Constraint Independent And/Or-Parallel System ['92-present]
  - Non-deterministic concurrent constraint language.
  - Subsumes Prolog, CLP, CC (+Andorra via transformation), ...
  - Distributed / net execution.

- Most Prolog systems have a notion of threads nowadays (SICStus, Ciao, SWI, Yap, XSB, B-Prolog), adequate for hand-coding coarse-grain parallelism.
Some comparison with work in other paradigms

- Much progress (e.g., in FORTRAN) for regular computations. But comparatively less on:
  - parallelization across procedure calls,
  - irregular computations,
  - complex data structures / pointers,
  - speculation, etc.
Wrap-up: (C)LP strong points

- Several generations of parallelizing compilers for LP and CLP [85-...]:
  - Good compilation speed, proved correct and efficient.
  - Speedups over state-of-the-art sequential systems on applications.
  - Good demonstrators of abstract interpretation as data-flow analysis technique.
  - Now including granularity control.

Improved on hand parallelizations on several large applications.

- Areas of particularly good progress:
  - Concepts of independence (pointers, search/speculation, constraints...).
  - Inter-procedural analysis (dynamic data, recursion, pointers/aliasing, etc.).
  - Parallelization algorithms for conditional dependency graphs.
  - Dealing with irregularity:
    - efficient task representation and fast dynamic scheduling,
    - static inference of task cost functions – granularity control.
  - Mixed static/dynamic parallelization techniques.
Wrap-up: areas for improvement

- Weaker areas / shortcomings:
  - In general, weak in detecting independence in structure traversals based on integer arithmetic (modeled as recursions over recursive data structures to fit parallelizer).
  - Weaker partitioning / placement for regular computations and static data structures.
  - Little work on mutating data structures (e.g., single assignment transformations).

- The objective is to perform all these tasks well also!

- Opportunities for synergy.

- A final plug for constraint programming:
  - Merges elegantly the symbolic and the numerical worlds.
  - We believe many of the features of CLP will make it slowly into mainstream languages (e.g., ILOG, ALMA, and other recent proposals).
Some general-purpose contributions from (C)LP

- Some examples so far:
  - Stealing-based scheduling strategies and microthreading.
  - Cactus-like stack memory management techniques.
  - Abstract interpretation-based static dependency analysis.
  - Sharing (aliasing) analyses, Shape analyses, ...
  - Parallelization (“annotation”) algorithms.
  - Cost analysis-based granularity control.
  - Logic variable-based synchronization.
  - Determinacy-based parallelization.
  - ...

M. Hermenegildo – Parallel Execution of Logic Programs

Compulog/ALP Summer School – Las Cruces, NM, July 24-27 2008
Some challenges?

- Parallelism not yet exploited on an everyday basis (real system, real applications).

- Some challenges:
  
  ◦ Scalability of techniques (from analysis to scheduling).
  
  ◦ Maintainability of the systems: simplification?
   
  * Move as much as possible to source level?
   
   (And explore this same route with many other things –e.g., tabling)

  ◦ Better automatic parallelization:
   
   * Better granularity control (e.g., time-based).
   * Better granularity-aware annotators.
   * Full scalability of analysis (modular analysis, etc.).
   * Automate program transformations (e.g., loop unrollings).

  ◦ Supporting multiple types of parallelism easily is still a challenge.

  ◦ A really elegant (and implementable) concurrent language which includes non-determinism.

  ◦ Combination w/low-level optimization and other features (r.g., or-// YapTab).
Some Bibliography (for a general tutorial see [GPA+01])


