Computational Logic

Concurrent (Constraint) Logic Programming
Concurrent Logic Programs

- **Predicate**: Set of clauses
- **Clause**: $Head \leftarrow Guard \mid Body$.
  - $Head$ is an atom
  - $Guard$ and $Body$ are conjunctions of atoms
- **Resolvent**: Set of goals (instances of atoms)
- **Operational semantics**: rewrite a goal in the resolvent with one of the clauses in the matching predicate definition
- **Concurrency**:
  - “No” goal selection rule (i.e., concurrent selection rule)
  - “No” clause search rule (i.e., concurrent search rule)
Synchronization Rules

- Clause matching: $Head + Guard$.
  - $Head$ matches the goal
  - $Guard$ is successful
- A head matches a goal if the goal is an instance of the head
- A guard is executed in one-way unification mode
- Suspension: if a head does not match the goal, but it could do so in the future, then it suspends
An Example

\[
\begin{align*}
p(X) & \text{:- } X = a \mid r. \\
p(X) & \text{:- } X = b \mid s. \\
q(X) & \text{:- true} \mid X = b.
\end{align*}
\]

?- p(X), q(X).

- There is no ordering in the execution of \( \langle p(X), q(X) \rangle \)
- There is no ordering in the execution of clauses of \( p(X) \)
- Clauses of \( p(X) \) suspend
- The clause of \( q(X) \) continues ("commits")
- Then, \( q(X) \) instantiates \( \{X/b\} \) in the body
- The second clause of \( p(X) \) continues ("commits"), while first clause fails.
Logic vs. Concurrent Logic Programming

- The logical variable as a communication channel

<table>
<thead>
<tr>
<th>Logic</th>
<th>Concurrent Logic</th>
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<tr>
<td>shared logical variable</td>
<td>communication channel</td>
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<tr>
<td>instantiation</td>
<td>communication</td>
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<tr>
<td>head unification</td>
<td>synchronization</td>
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</table>

- **Unification Revisited:**
  - **One-way (Read-only) unification — Ask**
    - *in Head and in Guard*
  - **Two-way (Output) unification — Tell**
    - *only in Body*
  - **Suspension:**
    - *Due to read-only unification in clause selection*
Logic vs. Concurrent Logic Programming

- Committed-choice: clause selection is irrevocable
- No backtracking allowed

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<th>Concurrent Logic</th>
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<tbody>
<tr>
<td>cut</td>
<td>commit</td>
</tr>
<tr>
<td>“don’t know” non-determinism</td>
<td>(“don’t care” non-determinism) indeterminism</td>
</tr>
<tr>
<td>search</td>
<td>selection</td>
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</table>

- Guards:
  - Flat guards: only selected predicates in guards
    * (Special) builtins
    * Possibly also facts
  - Deep guards: calls to any predicate allowed in guards
    * User-defined predicates, too
Logic vs. Concurrent Logic Programming

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<tr>
<td>atomic goal</td>
<td>process</td>
</tr>
<tr>
<td>goal (set of atoms)</td>
<td>process network</td>
</tr>
<tr>
<td>clause</td>
<td>process instruction</td>
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- Goals as processes:

- Process Behaviour:
  - Change state of process network:
    - * Become a new process:
    - * Become $k$ concurrent processes:
  - Halt:
  - Change state of data:

- Some syntactic sugar:
  - $A \triangleright G \mid true \Leftrightarrow A \triangleright G \mid \cdot$
  - $A \triangleright true \mid G \Leftrightarrow A \triangleright \mid G \Leftrightarrow A \triangleright G$
  - $A \triangleright true \mid true \Leftrightarrow A$
Process Behaviour Examples

- Become a new process: \( A :- G | B. \)
  
  \[ p(X) :- X = f(a,Y) \mid q(Y). \]

- Become \( k \) concurrent processes: \( A :- G | B_1...B_k. \)
  
  \[ p(X) :- X = f(A,B,C) \mid q(A), r(B), s(C). \]

- Halt: \( A :- G | . \)
  
  \[ p(X) :- X = f(a) \mid . \]

- Change state of data: \( A :- G | ...A. \)
  
  \[ p(X) :- X = f(a,Y) \mid Y = f(b,Z), p(Z). \]
  \[ p(I,S) :- I = [H|NI], \text{int}(H) \mid NS \text{ is } S+H, p(NI,NS). \]
Incomplete Messages

- Back-communication:

  \[?- q(X), p(X).\]

  \[p(X) :- X=f(a,Y), \text{check}(Y).\]

  \[\text{check}(\text{ok}).\]

  \[q(f(X,Y)) :- X=a \mid Y=\text{ok}.\]
Incomplete Messages (Contd.)

- Dialogue:

  \[ - q(X), p(\text{more}(X)). \]

  \[
p(\text{more}(X)) : - X = f(a, Y), p(Y).
p(\text{more}(X)) : - X = f(b, Y), p(Y).
p(\text{ok}).
\]

  \[
q(f(X, Y)) : - X = b \mid Y = \text{more}(Z), q(Z).
q(f(X, Y)) : - X = a \mid Y = \text{ok}.
\]

- Network formation and reconfiguration:

  \[ - q(A), p(A). \]

  \[
p(A) : - A = \text{channels}(X, Y, Z), p1(X), p2(Y), p3(Z).
q(\text{channels}(X, Y, Z)) : - q1(X), q2(Y), q3(Z).
\]
The Logical Variable

- A shared variable acts like:
  - A communication channel to send a message
  - A shared location being accessed concurrently

- Equivalences/conceptual view:
  - One shared variable = One message
  - Instantiation = Sending a message
  - Partially instantiated term = incomplete message = open channel
  - Ground term = complete message = closed channel
  - Recursive term = stream of messages

- Incomplete structures: an incomplete message can be thought of as:
  - A message being incrementally sent
  - An open communication channel
  - A message with sender’s identity
  - A structure being co-operatively constructed
Streams of Messages

- A stream producer
  
  \[
  \text{naturals}(N, Is) :\quad \text{Is} = [N|I_{s1}], \quad N_1 \text{ is } N+1, \quad \text{naturals}(N_1, I_{s1}).
  \]

- A stream consumer
  
  \[
  \text{sum}([N|I_s], T_{mp}, S_{um}) :\quad N \geq 0 \quad \Rightarrow \quad T_{mp} = T_{mp} + N, \quad \text{sum}(I_{s}, T_{mp}, S_{um}).
  \]

- Producer/Consumer (asynchronous)
  
  \[
  \text{?- naturals}(0, I), \quad \text{sum}(I, 0, \text{Total}).
  \]

- Producer/Consumer on demand (synchronous)
  
  \[
  \text{?- naturals}(0, I), \quad \text{sum}(I, 0, \text{Total}), \quad I = [\_|\_].
  \]
  
  \[
  \text{naturals}(N, [I|I_s]) :\quad I = N, \quad N_1 \text{ is } N+1, \quad \text{naturals}(N_1, I_{s}).
  \]
  
  \[
  \text{sum}([N|I_s], T_{mp}, S_{um}) :\quad N \geq 0 \quad \Rightarrow \quad I_s = [\_|\_], \quad T_{mp} = T_{mp} + N, \quad \text{sum}(I_{s}, T_{mp}, S_{um}).
  \]

- Key issue: who produces the buffer?
Merging and Dispatching Streams

- A stream merger:
  \[
  \text{merge([X|Xs],Ys,Out):= Out=[X|Zs], merge(Xs,Ys,Zs).}
  \]
  \[
  \text{merge(Xs,[Y|Ys],Out):= Out=[Y|Zs], merge(Xs,Ys,Zs).}
  \]
  \[
  \text{merge([],Ys,Out):= Out=Ys.}
  \]
  \[
  \text{merge(Xs, [], Out):= Out=Xs.}
  \]

- A (copying) stream dispatcher?
  \[
  \text{dispatch([X|Xs],Out1,Out2):- Out1=[X|Ys], Out2=[X|Zs], dispatch(Xs,Ys,Zs).}
  \]
  \[
  \text{dispatch([],Out1,Out2):- Out1=[], Out2=[].}
  \]

- A (caotic) stream dispatcher:
  \[
  \text{dispatch([X|Xs],Out1,Out2):- Out1=[X|Ys], dispatch(Xs,Ys,Out2).}
  \]
  \[
  \text{dispatch([X|Xs],Out1,Out2):- Out2=[X|Ys], dispatch(Xs,Out1,Ys).}
  \]
  \[
  \text{dispatch([],Out1,Out2):- Out1=[], Out2=[].}
  \]

- A stream dispatcher with senders’ identities
  \[
  \text{dispatch([mess(1,X)|Xs],Out1,Out2):- Out1=[X|Ys], dispatch(Xs,Ys,Out2).}
  \]
  \[
  \text{dispatch([mess(2,X)|Xs],Out1,Out2):- Out2=[X|Ys], dispatch(Xs,Out1,Ys).}
  \]
  \[
  \text{dispatch([],Out1,Out2):- Out1=[], Out2=[].}
  \]
Fairness

“An event that may occur will eventually occur”

- Or-Indeterminism: clause selection $\Rightarrow$ Or-Fairness (clauses eventually selected)
- And-Indeterminism: goal reduction $\Rightarrow$ And-Fairness (allows non-terminating procs.)
- A stream merger:

  ```prolog
  merge([X|Xs],Ys,Out):- Out=[X|Zs], merge(Xs,Ys,Zs).
  merge(Xs,[Y|Ys],Out):- Out=[Y|Zs], merge(Xs,Ys,Zs).
  merge([],Ys,Out):- Out=Ys.
  merge(Xs,[],Out):- Out=Xs.
  ```

  Key: or-fairness required, otherwise it is just append!

- An eager producer:

  ```prolog
  naturals(N,Is):- | Is=[N|Is1], N1 is N+1, naturals(N1,Is1).
  ```

  ```prolog
  %- naturals(0,I), sum(I,0,Total).
  ```

  Key: and-fairness required, otherwise nothing is ever consumed!
Termination Issues

- Non–terminating (but running) processes:

?– naturals(I), sum(I,Total), I=[_|_].

naturals(I):- naturals(0,I).

naturals(N,[I|Is]):- I=N, N1 is N+1, naturals(N1,Is).

sum(I,Total):- sum(I,0,Total).

sum([N|Is],Tmp,Sum):- N>=0 | Is=[_|_], TN is Tmp+N, sum(Is,TN,Sum).
• Deadlock:

?- q(X), p(X).

p(more(X)) :- X=f(a,Y), p(Y).
p(more(X)) :- X=f(b,Y), p(Y).
p(ok).

q(f(X,Y)) :- X=b | Y=more(Z), q(Z).
q(f(X,Y)) :- X=a | Y=ok.
Bounded-Size Communication Media

- Producer/Consumer with fixed sized communication (e.g., size=4) and termination:

  ?- naturals(0,I), sum(I,0,Total), I=[_1,_2,_3,_4].

  naturals(N,[I|Is]):- | I=N, N1 is N+1, naturals(N1,Is).
  naturals(N,[]).

  sum([N|Is],Tmp,Sum):- N>=0 | TN is Tmp+N,sum(Is,TN,Sum).
  sum([],Tmp,Sum):- | Sum=Tmp.

  Key: the communication media is produced from outside and fixed size!

- Dynamically-sized media:

  ?- naturals(0,I), sum(I,0,Total), medium(4,I).

  medium(0,Stream) :- Stream = [].
  medium(N,Stream):- N>0 | Stream=[_|Stream1], medium(N-1,Stream1).
Bounded-Buffer Communication

- Bounded buffer:
  \[
  \text{buffer}(0, \text{Stream}, \text{Tail}) :- \quad \text{Stream} = \text{Tail}.
  \]
  \[
  \text{buffer}(N, \text{Stream}, \text{Tail}) :- \quad N > 0 \quad \mid \quad \text{Stream} = [\_ | \text{Stream1}], \quad \text{buffer}(N-1, \text{Stream1}, \text{Tail}).
  \]
  Creates buffer as open list of \( N \) elements, passes handle to list end

- Simple producer with termination at \( \text{Max} \) elements:
  \[
  \text{naturals}(N, [I | \text{Is}], \text{Max}) :- \quad N \leq \text{Max} \quad \mid \quad I = N, \quad \text{N1 is N+1}, \quad \text{naturals}(N1, \text{Is}, \text{Max}).
  \]
  \[
  \text{naturals}(N, I, \text{Max}) :- \quad N > \text{Max} \quad \mid \quad I = \[].
  \]
  Suspended until buffer available. Closes buffer at \( \text{Max} \) elements

- Consumer:
  \[
  \text{sum}([N | \text{Is}], \text{Tail}, \text{Acc}, \text{Sum}) :- \quad N \geq 0 \quad \mid \quad \text{Tail} = [\_ | \text{Tail1}], \quad \text{NAcc is Acc+N}, \quad \text{sum}(\text{Is}, \text{Tail1}, \text{NAcc}, \text{Sum}).
  \]
  \[
  \text{sum}([], \text{Tail}, \text{Acc}, \text{Sum}) :- \quad \text{Acc} = \text{Sum}.
  \]
  Suspended until buffer and element available. Adds one more element to the buffer for each element consumed.

- Usage (e.g., for buffer length = 18, termination at 1000 elements):
  \[
  ?- \quad \text{naturals}(0, \text{Buffer}, 1000), \quad \text{sum}(\text{Buffer}, \text{Tail}, 0, \text{Total}), \quad \text{buffer}(18, \text{Buffer}, \text{Tail}).
  \]

• Overall effect is still asynchronous!

• Producer can get ahead of consumer by a fixed number of elements. After that, suspended on stream until Consumer requests more.
Streams of Messages: Protocols

- One-to-one communication:  
  One producer + One consumer

- Duplex communication:  
  Two producer/consumers

- Broadcast communication:  
  One producer + Many consumers

- Many-to-one communication:  
  Many producers + One consumer

- Blackboard communication:  
  Many producers + Many consumers:  
  Many producers/consumers
Broadcast Communication

- Matrix multiplication:

\[\text{?- vector}(V), \text{matrix}(M), \text{vm}(V,M,\text{Result}).\]

\[
\text{vm}(_{\text{\_}}, [], Zv) :\text{- Zv} = [].
\]

\[
\text{vm}(Xv, [Yv | Ym], Zv) :\text{- Zv} = [Z | Zv1],
\]

\[
\text{vv}(Xv, Yv, Z),
\]

\[
\text{vm}(Xv, Ym, Zv1).
\]

\[
\text{vv}(Xv, Yv, P) :\text{- vv1}(Xv, Yv, 0, P).
\]

\[
\text{vv1}([], [], S, P) :\text{- P} = S.
\]

\[
\text{vv1}([X | Xv], [Y | Yv], S, P) :\text{- S1} \text{ is}\ S + X \times Y \mid
\]

\[
\text{vv1}(Xv, Yv, S1, P).
\]

- Broadcasting of V to all vv/3 processes
- Dynamically configured network of vv/3 processes
Many-to-one Communication

- A data abstraction: queues

  \[
  \text{queue}([\text{dequeue}(X)|S], \text{Head}, \text{Tail}): -
  \quad \text{Head}=[X|\text{NewHead}],
  \quad \text{queue}(S, \text{NewHead}, \text{Tail}).
  \]

  \[
  \text{queue}([\text{enqueue}(X)|S], \text{Head}, \text{Tail}): -
  \quad \text{Tail}=[X|\text{NewTail}],
  \quad \text{queue}(S, \text{Head}, \text{NewTail}).
  \]

  \[
  \text{queue}([], _, _, _).
  \]
• A simulator of a multiprocessor machine

```prolog
?- processors(10,Job), Job=...

processors(N,X):-
    queue(S,[X|Xs],Xs),
    processors(1,N,S).

processors(N,N,S):-
    processor(N,idle,S).

processors(N1,N4,S):-
    N2 is (N1+N4)/2 | N3 is N2+1,
    processors(N1,N2,S1),
    processors(N3,N4,S2),
    merge(S1,S2,S).
```

• N processor/3 proc. communicating with one queue/3 proc.

• Statically configured network of proc.: spawning / computing phases ("systolic")
Many-to-many Communication

- A network of producers and consumers

```prolog
?- consumers(Buffer), producers(Buffer).

producers(Stream):- p1(X), p2(Y), p3(Z),
    merge(X,Y,Stream1), merge(Z,Stream1,Stream).

consumers(Stream):- c1(Stream), c2(Stream), c3(Stream).

p1(S):- S=[message(1,Mess)|Xs], produce(Mess), p1(Xs).
p1(S):- S=[].

c1([X|Xs]):- X=message(1,Mess) | consume(Mess), c1(Xs).
c1([X|Xs]):- X=message(Id,Mess), Id=\=1 | c1(Xs).
c1([]).
```

- Blackboard Communication:
  - Needed driver for the blackboard
Operational Semantics

- Rewriting system

\[
\text{match}(A, A') = \begin{cases} 
\theta & \text{if } A = A'\theta \text{ and } \text{mgu}(A, A') = \theta \\
\text{fail} & \text{if } \text{mgu}(A, A') = \text{fail} \\
\text{suspend} & \text{otherwise}
\end{cases}
\]

\[
\text{try}(A, (A' \leftarrow G \mid B)) = \begin{cases} 
\theta & \text{if } \text{match}(A, A') = \theta \land \\
& \text{check}(G\theta) = \text{true} \\
\text{fail} & \text{if } \text{match}(A, A') = \theta \land \\
& \text{check}(G\theta) = \text{fail} \lor \\
& \text{match}(A, A') = \text{fail} \\
\text{suspend} & \text{otherwise}
\end{cases}
\]
Operational Semantics (Contd.)

- **Reduction**: $A_1...A_i...A_n; \theta \rightarrow (A_1...B_1...B_k...A_n)\theta'; \theta \circ \theta'
  \text{if } \exists C = A \leftarrow G | B_1...B_n \text{ s.t. } try(A_i, C) = \theta'$

- **Failure**: $A_1...A_i...A_n; \theta \rightarrow fail; \theta$
  \text{if } \forall C \ try(A_i, C) = fail

- **Guard checking**:
  - Flat guards: use *match* in all unifications
  - Deep guards: copy environment
(Some) Concurrent Logic Languages

- **Parlog [Clark, Gregory 83]**
  - mode declarations for input/output arguments
  - safe clauses: output instantiation in guards is an error
  - one-way unification in guards

- **Concurrent Prolog [Shapiro 84]**
  - read-only annotation of variables in calls
  - local environments for guards
  - atomic extended head unification

- **GHC (Guarded Horn Clauses) [Ueda 85]**
  - different interpretation of unification in guard and body
  - suspension on output instantiation in guards
  - general unification with guard restriction
(Some) Concurrent Logic Languages (Contd.)

- Implementation Issues:
  - Parlog
    * compile-time safety check
  - Concurrent Prolog
    * support for local environments
    * detection of inconsistency with global environment
  - GHC
    * identification of variables on which to suspend

- Problems: no backtracking.

- More Recent Systems:
  - Andorra-I: only deterministic computations proceed.
  - AKL: goals execute in a local environment.
  - BinProlog: communication through blackboard.
  - CIAO: communication through shared database.