Computational Logic

Concurrent (Constraint) Logic Programming
Concurrent Logic Programs

- **Predicate**: Set of clauses
- **Clause**: $Head \ :- \ Guard \mid Body$.
  - $Head$ is an atom
  - $Guard$ and $Body$ are conjunctions of atoms
- **Resolvent**: Set of goals (instances of atoms)
- **Operational semantics**: rewrite a goal in the resolvent with one of the clauses in the matching predicate definition
- **Concurrency**:
  - “No” goal selection rule (i.e., concurrent selection rule)
  - “No” clause search rule (i.e., concurrent search rule)
Synchronization Rules

- Clause matching: $Head + Guard$.
  - $Head$ matches the goal
  - $Guard$ is successful

- A head matches a goal if the goal is an instance of the head

- A guard is executed in one-way unification mode

- Suspension: if a head does not match the goal, but it could do so in the future, then it suspends
An Example

\[
\begin{align*}
p(X) & : - X = a \mid r. \\
p(X) & : - X = b \mid s. \\
q(X) & : - \text{true} \mid X = b.
\end{align*}
\]

?- p(X), q(X).

• There is no ordering in the execution of \( \langle p(X), q(X) \rangle \)
• There is no ordering in the execution of clauses of \( p(X) \)
• Clauses of \( p(X) \) suspend
• The clause of \( q(X) \) continues ("commits")
• Then, \( q(X) \) instantiates \( \{X/b\} \) in the body
• The second clause of \( p(X) \) continues ("commits"), while first clause fails.
Logic vs. Concurrent Logic Programming

- The logical variable as a communication channel

<table>
<thead>
<tr>
<th>Logic</th>
<th>Concurrent Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>shared logical variable</td>
<td>communication channel</td>
</tr>
<tr>
<td>instantiation</td>
<td>communication</td>
</tr>
<tr>
<td>head unification</td>
<td>synchronization</td>
</tr>
</tbody>
</table>

- Unification Revisited:
  - One-way (Read-only) unification — Ask
    - in $Head$ and in $Guard$
  - Two-way (Output) unification — Tell
    - only in $Body$
  - Suspension:
    - Due to read-only unification in clause selection
Logic vs. Concurrent Logic Programming

- Committed-choice: clause selection is irrecoverable
- No backtracking allowed

<table>
<thead>
<tr>
<th>Logic</th>
<th>Concurrent Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>cut</td>
<td>commit</td>
</tr>
<tr>
<td>“don’t know” non-determinism</td>
<td>(“don’t care” non-determinism)</td>
</tr>
<tr>
<td>search</td>
<td>selection</td>
</tr>
</tbody>
</table>

- Guards:
  - **Flat** guards: only selected predicates in guards
    - (Special) builtins
    - Possibly also facts
  - **Deep** guards: calls to any predicate allowed in guards
    - User-defined predicates, too
Logic vs. Concurrent Logic Programming

- Goals as processes:

<table>
<thead>
<tr>
<th>Logic</th>
<th>Concurrent Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>atomic goal</td>
<td>process</td>
</tr>
<tr>
<td>goal (set of atoms)</td>
<td>process network</td>
</tr>
<tr>
<td>clause</td>
<td>process instruction</td>
</tr>
</tbody>
</table>

- Process Behaviour:
  - Change state of process network:
    - Become a new process:
    - Become $k$ concurrent processes:
  - Halt:
  - Change state of data:

- Some syntactic sugar:

  - $A ::= G \mid true. \
    \Leftrightarrow A ::= G \mid .$
  - $A ::= true \mid G. \Leftrightarrow A ::= \mid G. \Leftrightarrow A ::= G.$
  - $A ::= true \mid true. \Leftrightarrow A.$
Process Behaviour Examples

- Become a new process: $A ::= G | B$.
  
  $p(X) ::= X = f(a,Y) | q(Y)$. 

- Become $k$ concurrent processes: $A ::= G | B_1 ... B_k$.
  
  $p(X) ::= X = f(A,B,C) | q(A), r(B), s(C)$. 

- Halt: $A ::= G | .$
  
  $p(X) ::= X = f(a) | .$. 

- Change state of data: $A ::= G | ...A$.
  
  $p(X) ::= X = f(a,Y) | Y = f(b,Z), p(Z)$.  
  
  $p(I,S) ::= I = [H | NI], \text{int}(H) | NS \text{ is } S + H, p(NI,NS)$. 

Incomplete Messages

• Back-communication:

  \(?- q(X), p(X).

  p(X) :- X=f(a,Y), check(Y).

  check(ok).

  q(f(X,Y)) :- X=a \mid Y=ok.
Incomplete Messages (Contd.)

- **Dialogue:**

  \[
  ?- q(X), \ p(more(X)).
  \]

  \[
  p(more(X)):- X=f(a,Y), \ p(Y).
  \]

  \[
  p(more(X)):- X=f(b,Y), \ p(Y).
  \]

  \[
  p(ok).
  \]

  \[
  q(f(X,Y)):- X=b \ | \ Y=more(Z), \ q(Z).
  \]

  \[
  q(f(X,Y)):- X=a \ | \ Y=ok.
  \]

- **Network formation and reconfiguration:**

  \[
  ?- q(A), \ p(A).
  \]

  \[
  p(A):- A=channels(X,Y,Z), \ p1(X), \ p2(Y), \ p3(Z).
  \]

  \[
  q(channels(X,Y,Z)):- q1(X), \ q2(Y), \ q3(Z).
  \]
The Logical Variable

- A shared variable acts like:
  - A communication channel to send a message
  - A shared location being accessed concurrently

- Equivalences/conceptual view:
  - One shared variable = One message
  - Instantiation = Sending a message
  - Partially instantiated term = incomplete message = open channel
  - Ground term = complete message = closed channel
  - Recursive term = stream of messages

- Incomplete structures: an incomplete message can be thought of as:
  - A message being incrementally sent
  - An open communication channel
  - A message with sender’s identity
  - A structure being co-operatively constructed
Streams of Messages

- A stream producer
  \[\text{naturals}(N, Is) : - \ Is = [N|Is1], \ N1 \text{ is } N+1, \ \text{naturals}(N1, Is1)\].

- A stream consumer
  \[\text{sum}([N|Is], Tmp, Sum) : - \ N \geq 0 \ | \ TN \text{ is } Tmp+N, \ \text{sum}(Is, TN, Sum)\].

- Producer/Consumer (asynchronous)
  \[- \text{naturals}(0, I), \ \text{sum}(I, 0, Total)\].

- Producer/Consumer on demand (synchronous)
  \[- \text{naturals}(0, I), \ \text{sum}(I, 0, Total), \ I = [\_|\_]\].

\[\text{naturals}(N, [I|Is]) : - \ I = N, \ N1 \text{ is } N+1, \ \text{naturals}(N1, Is)\].

\[\text{sum}([N|Is], Tmp, Sum) : - \ N \geq 0 \ | \ Is = [\_|\_], \ TN \text{ is } Tmp+N, \ \text{sum}(Is, TN, Sum)\].

- Key issue: who produces the buffer?
Merging and Dispatching Streams

• A stream merger:
  
  - `merge([X|Xs],Ys,Out):- Out=[X|Zs], merge(Xs,Ys,Zs).`
  
  - `merge(Xs,[Y|Ys],Out):- Out=[Y|Zs], merge(Xs,Ys,Zs).`
  
  - `merge([],Ys,Out):- Out=Ys.`
  
  - `merge(Xs,[],Out):- Out=Xs.`

• A (copying) stream dispatcher?
  
  - `dispatch([X|Xs],Out1,Out2):- Out1=[X|Ys], Out2=[X|Zs], dispatch(Xs,Ys,Zs).`
  
  - `dispatch([],Out1,Out2):- Out1=[], Out2=[].`

• A (caotic) stream dispatcher:
  
  - `dispatch([X|Xs],Out1,Out2):- Out1=[X|Ys], dispatch(Xs,Ys,Out2).`
  
  - `dispatch([X|Xs],Out1,Out2):- Out2=[X|Ys], dispatch(Xs,Out1,Ys).`
  
  - `dispatch([],Out1,Out2):- Out1=[], Out2=[].`

• A stream dispatcher with senders’ identities
  
  - `dispatch([mess(1,X)|Xs],Out1,Out2):- Out1=[X|Ys], dispatch(Xs,Ys,Out2).`
  
  - `dispatch([mess(2,X)|Xs],Out1,Out2):- Out2=[X|Ys], dispatch(Xs,Out1,Ys).`
  
  - `dispatch([],Out1,Out2):- Out1=[], Out2=[].`
Fairness

“An event that may occur will eventually occur”

- Or-Indeterminism: clause selection $\Rightarrow$ Or-Fairness (clauses eventually selected)
- And-Indeterm.: goal reduction $\Rightarrow$ And-Fairness (allows non-terminating procs.)
- A stream merger:

  ```prolog
  merge([X|Xs], Ys, Out):- Out=[X|Zs], merge(Xs, Ys, Zs).
  merge(Xs, [Y|Ys], Out):- Out=[Y|Zs], merge(Xs, Ys, Zs).
  merge([], Ys, Out):- Out=Ys.
  merge(Xs, [], Out):- Out=Xs.
  ```

  Key: or-fairness required, otherwise it is just append!

- An eager producer:

  ```prolog
  naturals(N, Is):- | Is=[N|Is1], N1 is N+1, naturals(N1, Is1).
  ```

  ?- naturals(0, I), sum(I, 0, Total).

  Key: and-fairness required, otherwise nothing is ever consumed!
Termination Issues

• Non–terminating (but running) processes:

```prolog
?- naturals(I), sum(I,Total), I=\[_\|\_\].

naturals(I):- naturals(0,I).

naturals(N,[I|Is]):- I=N, N1 is N+1, naturals(N1,Is).

sum(I,Total):- sum(I,0,Total).

sum([N|Is],Tmp,Sum):- N>=0 | Is=\[_\|\_\], TN is Tmp+N, sum(Is,TN,Sum).
```
Termination Issues (Contd.)

- Deadlock:

```prolog
?- q(X), p(X).

p(more(X)):- X=f(a,Y), p(Y).
p(more(X)):- X=f(b,Y), p(Y).
p(ok).

q(f(X,Y)):- X=b | Y=more(Z), q(Z).
q(f(X,Y)):- X=a | Y=ok.
```
Bounded-Size Communication Media

- Producer/Consumer with fixed sized communication (e.g., size=4) and termination:

\[
\text{?- naturals}(0,I), \text{sum}(I,0,\text{Total}), I=[ \_1, \_2, \_3, 4].
\]

\[
\text{naturals}(N,[I|Is]):- | I=N, N1 \text{ is } N+1, \text{naturals}(N1,Is).
\]

\[
\text{naturals}(N,[]).
\]

\[
\text{sum}([N|Is],\text{Tmp},\text{Sum}):- N>=0 \text{ | } TN \text{ is } Tmp+N,\text{sum}(Is,TN,\text{Sum}).
\]

\[
\text{sum}([],\text{Tmp},\text{Sum}):- | \text{Sum}=\text{Tmp}.
\]

Key: the communication media is produced from outside and fixed size!

- Dynamically-sized media:

\[
\text{?- naturals}(0,I), \text{sum}(I,0,\text{Total}), \text{medium}(4,I).
\]

\[
\text{medium}(0,\text{Stream}) :- \text{Stream} = [].
\]

\[
\text{medium}(N,\text{Stream}) :- N>0 \text{ | } \text{Stream}=[ \_|\text{Stream1}], \text{medium}(N-1,\text{Stream1}).
\]
Bounded-Buffer Communication

- Bounded buffer:
  \[\text{buffer}(0, \text{Stream}, \text{Tail}) :- \text{Stream} = \text{Tail}.\]
  \[\text{buffer}(N, \text{Stream}, \text{Tail}) :- N > 0 \mid \text{Stream} = [\_ | \text{Stream}_1], \text{buffer}(N - 1, \text{Stream}_1, \text{Tail}).\]

  Creates buffer as open list of N elements, passes handle to list end

- Simple producer with termination at Max elements:
  \[\text{naturals}(N, [I | \text{Is}], \text{Max}) :- N \leq \text{Max} \mid I = N, N1 \text{ is } N + 1, \text{naturals}(N1, \text{Is}, \text{Max}).\]
  \[\text{naturals}(N, I, \text{Max}) :- N > \text{Max} \mid I = [].\]

  Suspended until buffer available. Closes buffer at Max elements

- Consumer:
  \[\text{sum}([N | \text{Is}], \text{Tail}, \text{Acc}, \text{Sum}) :- N \geq 0 \mid\]
  \[\text{Tail} = [\_ | \text{Tail}_1], \text{NAcc} \text{ is } \text{Acc} + N, \text{sum}(	ext{Is}, \text{Tail}_1, \text{NAcc}, \text{Sum}).\]
  \[\text{sum}([], \text{Tail}, \text{Acc}, \text{Sum}) :- \text{Acc} = \text{Sum}.\]

  Suspended until buffer and element available. Adds one more element to the buffer for each element consumed.

- Usage (e.g., for buffer length = 18, termination at 1000 elements):
  \[?- \text{naturals}(0, \text{Buffer}, 1000), \text{sum}(	ext{Buffer}, \text{Tail}, 0, \text{Total}), \text{buffer}(18, \text{Buffer}, \text{Tail}).\]
Bounded-Buffer Communication (Contd.)

- Overall effect is still asynchronous!
- Producer can get ahead of consumer by a fixed number of elements. After that, suspended on stream until Consumer requests more.
Streams of Messages: Protocols

- One-to-one communication:
  One producer + One consumer

- Duplex communication:
  Two producer/consumers

- Broadcast communication:
  One producer + Many consumers

- Many-to-one communication:
  Many producers + One consumer

- Blackboard communication:
  Many producers + Many consumers:
  Many producers/consumers
Broadcast Communication

- Matrix multiplication:

  \[ \text{vector}(V), \text{matrix}(M), \text{vm}(V,M,\text{Result}). \]

  \[
  \text{vm}(\_,[\_],Zv):- Zv=[\_].
  \]

  \[
  \text{vm}(Xv,[Yv|Ym],Zv):- Zv=[Z|Zv1],
  \text{vv}(Xv,Yv,Z),
  \text{vm}(Xv,Ym,Zv1).
  \]

  \[
  \text{vv}(Xv,Yv,P):- \text{vv1}(Xv,Yv,0,P).
  \]

  \[
  \text{vv1}([],[],S,P):- P=S.
  \]

  \[
  \text{vv1}([X|Xv],[Y|Yv],S,P):- S1 \text{ is } S+X\times Y \mid
  \text{vv1}(Xv,Yv,S1,P).
  \]

- Broadcasting of \( V \) to all \( vv/3 \) processes

- Dynamically configured network of \( vv/3 \) processes
Many-to-one Communication

- A data abstraction: queues

  
  queue([dequeue(X)|S],Head,Tail):-
  Head=[X|NewHead],
  queue(S,NewHead,Tail).

  queue([enqueue(X)|S],Head,Tail):-
  Tail=[X|NewTail],
  queue(S,Head,NewTail).

  queue([],_,_).

  

Many-to-one Communication (Contd.)

- A simulator of a multiprocessor machine

```
?- processors(10,Job), Job=...

processors(N,X):-
    queue(S,[X|Xs],Xs),
    processors(1,N,S).

processors(N,N,S):-
    processor(N,idle,S).
processors(N1,N4,S):-
    N2 is (N1+N4)/2 | N3 is N2+1,
    processors(N1,N2,S1),
    processors(N3,N4,S2),
    merge(S1,S2,S).
```

- N processor/3 proc. communicating with one queue/3 proc.
- Statically configured network of proc.: spawning / computing phases ("systolic")
Many-to-many Communication

- A network of producers and consumers

```prolog
?- consumers(Buffer), producers(Buffer).

producers(Stream):- p1(X), p2(Y), p3(Z),
    merge(X,Y,Stream1), merge(Z,Stream1,Stream).

consumers(Stream):- c1(Stream), c2(Stream), c3(Stream).

p1(S):- S=[message(1,Mess)|Xs], produce(Mess), p1(Xs).
p1(S):- S=[].

c1([X|Xs]):- X=message(1,Mess) | consume(Mess), c1(Xs).
c1([X|Xs]):- X=message(Id,Mess), Id\=1 | c1(Xs).
c1([]).
```

- Blackboard Communication:
  - Needed driver for the blackboard
Operational Semantics

- Rewriting system

\[
match(A, A') = \begin{cases} 
\theta & \text{if } A = A'\theta \text{ and } \text{mgu}(A, A') = \theta \\
\text{fail} & \text{if } \text{mgu}(A, A') = \text{fail} \\
\text{suspend} & \text{otherwise}
\end{cases}
\]

\[
\text{try}(A, (A' \leftarrow G \mid B)) = \begin{cases} 
\theta & \text{if } \text{match}(A, A') = \theta \land \\
\text{fail} & \text{if } \text{match}(A, A') = \theta \land \\
\text{suspend} & \text{otherwise}
\end{cases}
\]

\[
\begin{aligned}
\text{check}(G\theta) = \text{true} \\
\text{check}(G\theta) = \text{fail} \lor \\
\text{match}(A, A') = \text{fail}
\end{aligned}
\]
Operational Semantics (Contd.)

- **Reduction:** \( A_1...A_i...A_n; \theta \rightarrow (A_1...B_1...B_k...A_n)\theta'; \theta \circ \theta' \)
  
  if \( \exists C = A \leftarrow G \mid B_1...B_n \) s.t. \( \text{try}(A_i, C) = \theta' \)

- **Failure:** \( A_1...A_i...A_n; \theta \rightarrow fail; \theta \)
  
  if \( \forall C \ \text{try}(A_i, C) = fail \)

- **Guard checking:**
  - Flat guards: use \textit{match} in all unifications
  - Deep guards: copy environment
(Some) Concurrent Logic Languages

- **Parlog [Clark, Gregory 83]**
  - mode declarations for input/output arguments
  - safe clauses: output instantiation in guards is an error
  - one-way unification in guards

- **Concurrent Prolog [Shapiro 84]**
  - read-only annotation of variables in calls
  - local environments for guards
  - atomic extended head unification

- **GHC (Guarded Horn Clauses) [Ueda 85]**
  - different interpretation of unification in guard and body
  - suspension on output instantiation in guards
  - general unification with guard restriction
• Implementation Issues:
  ◇ Parlog
    * compile-time safety check
  ◇ Concurrent Prolog
    * support for local environments
    * detection of inconsistency with global environment
  ◇ GHC
    * identification of variables on which to suspend

• Problems: no backtracking.

• More Recent Systems:
  ◇ Andorra-I: only deterministic computations proceed.
  ◇ AKL: goals execute in a local environment.
  ◇ BinProlog: communication through blackboard.
  ◇ CIAO: communication through shared database.